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AN IN-DEPTH PROBABILISTIC STUDY OF EXTERNAL TANK ATTACH RING

By F. Pizzano and C. Putcha

System Safety and Reliability Office

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13. ABSTRACT *(Maximum 200 words)*

This report deals with conducting a probabilistic study of the external tank attach ring (ETA) used as an interface between the external tank attach struts and the solid rocket booster. The ideas was to use probabilistic distributions for material, geometric, and load properties, to calculate probabilistic margins of safety, and then to compare results against the deterministic factors of safety that were used in the actual design process. The report describes how this was done and discusses some of the road blocks and data problems that were encountered during the study and provides some conclusions. A further refinement of this study is being considered for future work which would make more direct use of finite element analysis data coupled with Monte Carlo simulation. The basic conclusion herein indicates that the probabilistic margins of safety for the cases analyzed (by use of existing data) appear to support deterministic results and actually indicate higher reliabilities.

14. SUBJECT TERMS

external tank attach (ETA) ring, probabilistic margin of safety (PMS), solid rocket booster (SRB), Monte Carlo simulation, finite element analysis

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TECHNICAL MEMORANDUM

AN IN-DEPTH PROBABILISTIC STUDY OF EXTERNAL TANK ATTACH RING

I. INTRODUCTION

This work is part of an overall study in the area of evaluating the performance of the shuttle. Besides the orbiter and space shuttle main engines (SSME's), the other main items of a shuttle are the solid rocket boosters (SRB's), external tank (ET), and the ET attachment (ETA) ring. The present study is restricted to the ETA, but the concepts discussed are general in nature and can be applied to any structural component. The strengths of steel, the cross-sectional dimensions of the steel section, and thus the strength of the model itself and the loads are all variable parameters. In other words, taking the entire structural element under consideration, the resistance and the applied loads are random variables. Consequently, the traditionally used deterministic analysis approach (the so-called factor of safety) does not reveal the actual safety reserve in the structure, because it does not include the inherent variability in the material, geometric, and load parameters. Such a variability can easily be included in the probabilistic analyses using probabilistic methods which have been in use and continually improved over the last two decades. The objective of this research work is to use some of the existing probabilistic methods to calculate the reliability of the ETA ring at various critical sections for the limit state of stress. This is done both in terms of the traditional probability of failure (p_f) and reliability levels as well as the well-known safety indices (β) which have become a commonly accepted measure of safety.

For any probabilistic analysis, a certain deterministic model is needed for which the limit state function can be formulated in terms of the general basic variables of resistance and load. The variability of the materials, cross-sectional properties, and load can then be incorporated into the deterministic model, and the β value can be calculated which forms the basis of the probabilistic model. The detail work of the deterministic model and the corresponding deterministic analysis were done by United Space Boosters, Inc., (USBI)[1] under a general contract to Marshall Space Flight Center (MSFC) and form a basis for the present probabilistic study.

II. DETERMINISTIC MODEL

The ETA ring used in the present analyses is part of the solid rocket aft booster assembly and is located at SRB station 1511.0. It is the interface between the ET attach struts and the SRM. The ETA ring is shown in figure 1. It extends a full 360° around the SRM case, and its cross section is shown in figure 2. The details of the ETA ring are given elsewhere.[2] The pertinent details needed for the probabilistic study (discussed later in this report) are briefly dealt with in this section. The primary purpose of the ETA ring is the distribution of concentrated attach strut loads circumferentially into the SRM case. The strut attachment is a pinned connection designed to react loads in the plane of the ring. Secondary out-of-plane loading exists if the struts are misaligned. Four splices are required for the ETA ring configuration in figure 1. A typical spliced section is shown in figure 3. The outer systems tunnel splice plate is 4340 steel, heat treated to 180 lb/in^2 minimum tensile strength. The inner splice plate is 4130 steel, heat treated to 180 lb/in^2 minimum strength.

The critical sections of the deterministic model of the ETA ring are found to be the tunnel splice and H-fitting.[1] These are shown in figures 4 and 5. Figure 4 shows the strut fitting, and figure 5 shows the tunnel splice.

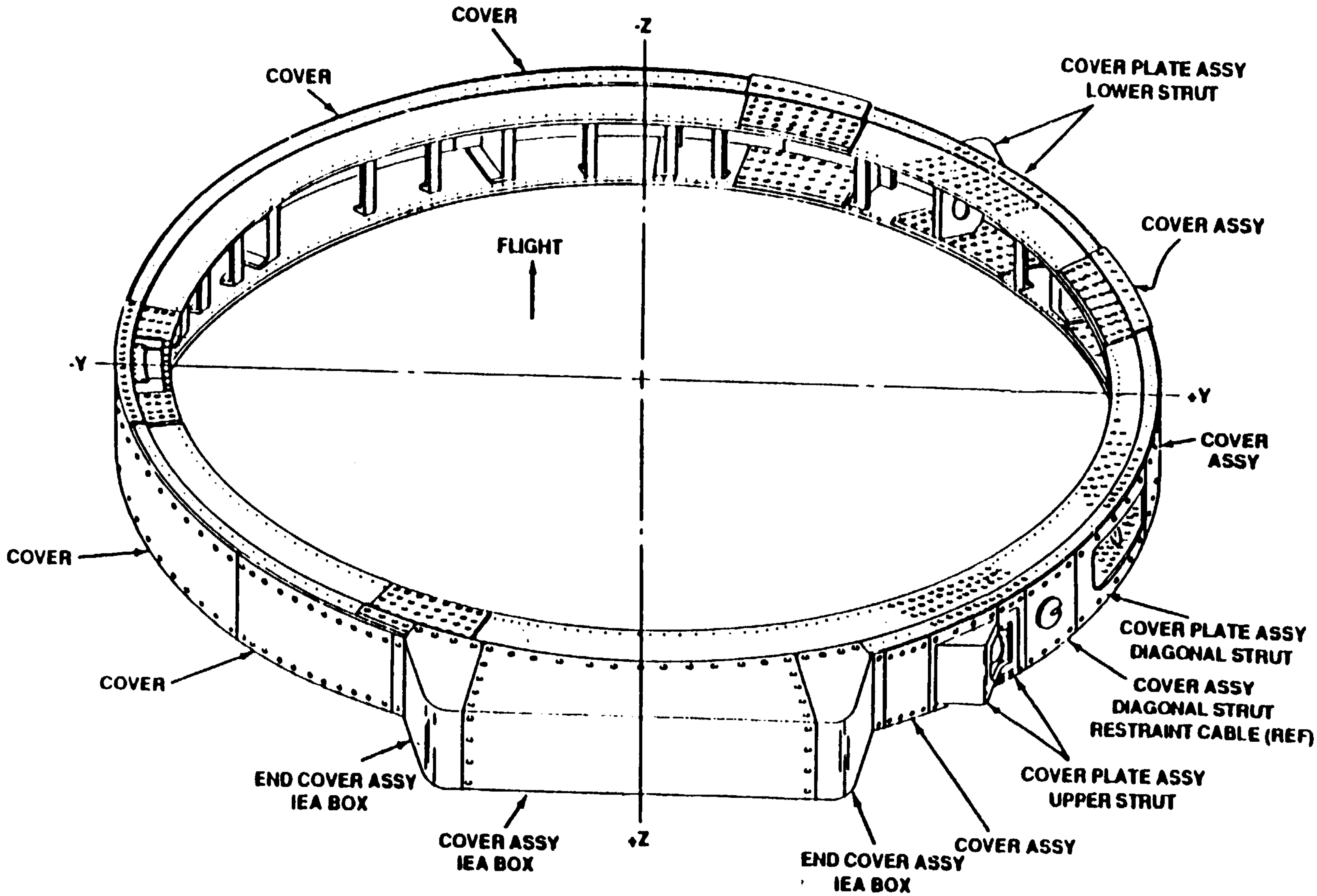


Figure 1. SRB/ET attach ring (insulation omitted for clarity) (from ref. 1).

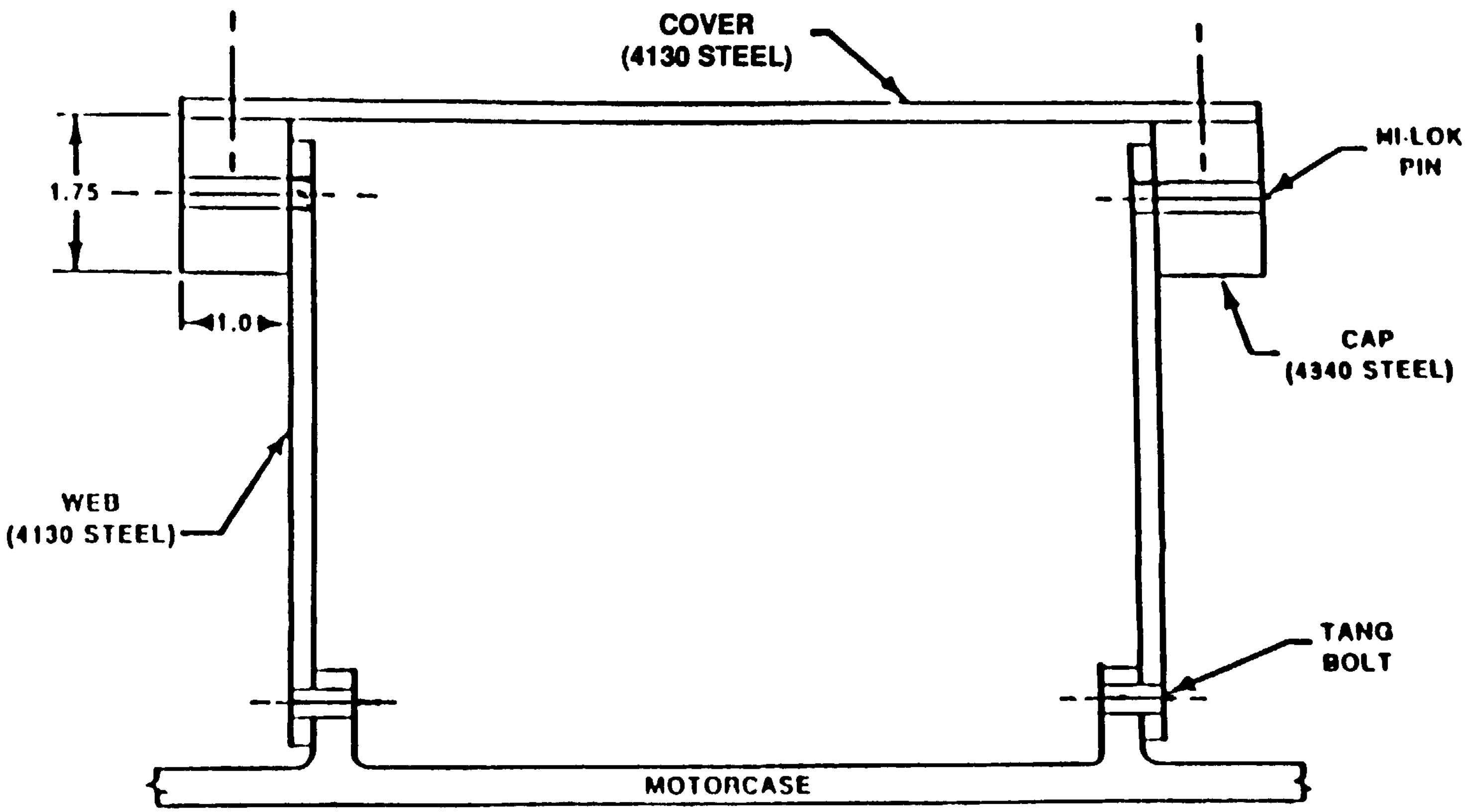


Figure 2. ETA ring cross section (from ref 1).

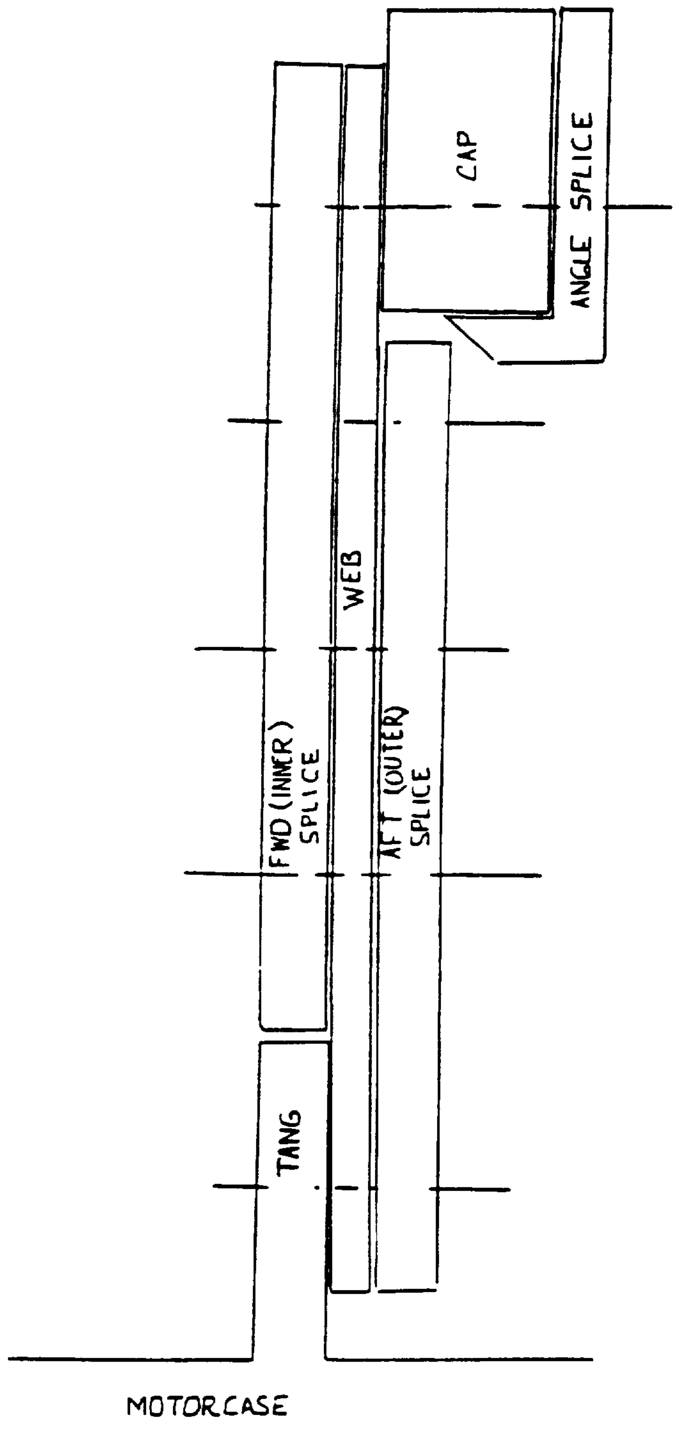


Figure 3. ETA ring splice section at 154°, 289°, and 342° (from ref. 1).

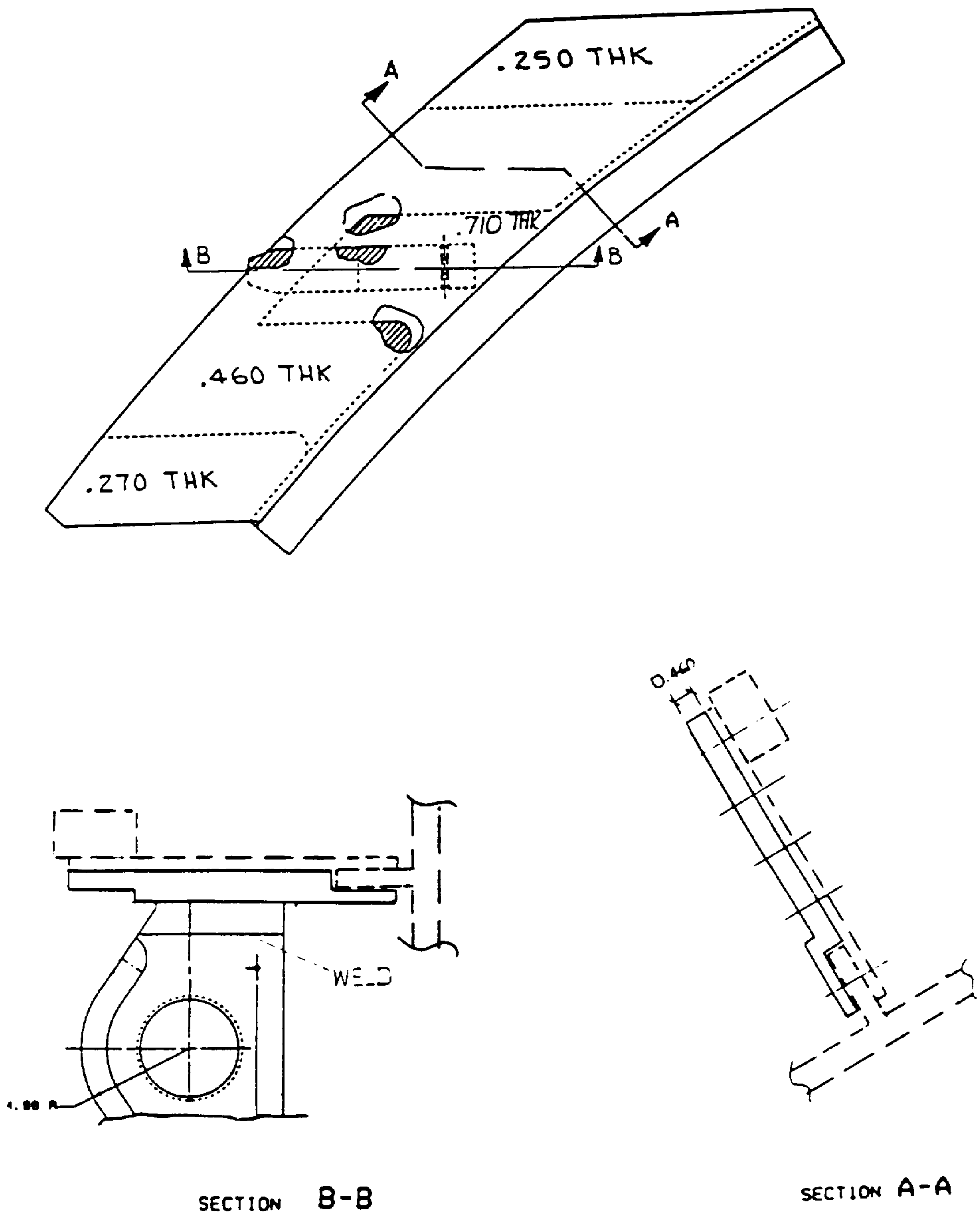


Figure 4. Strut fitting (from ref. 1).

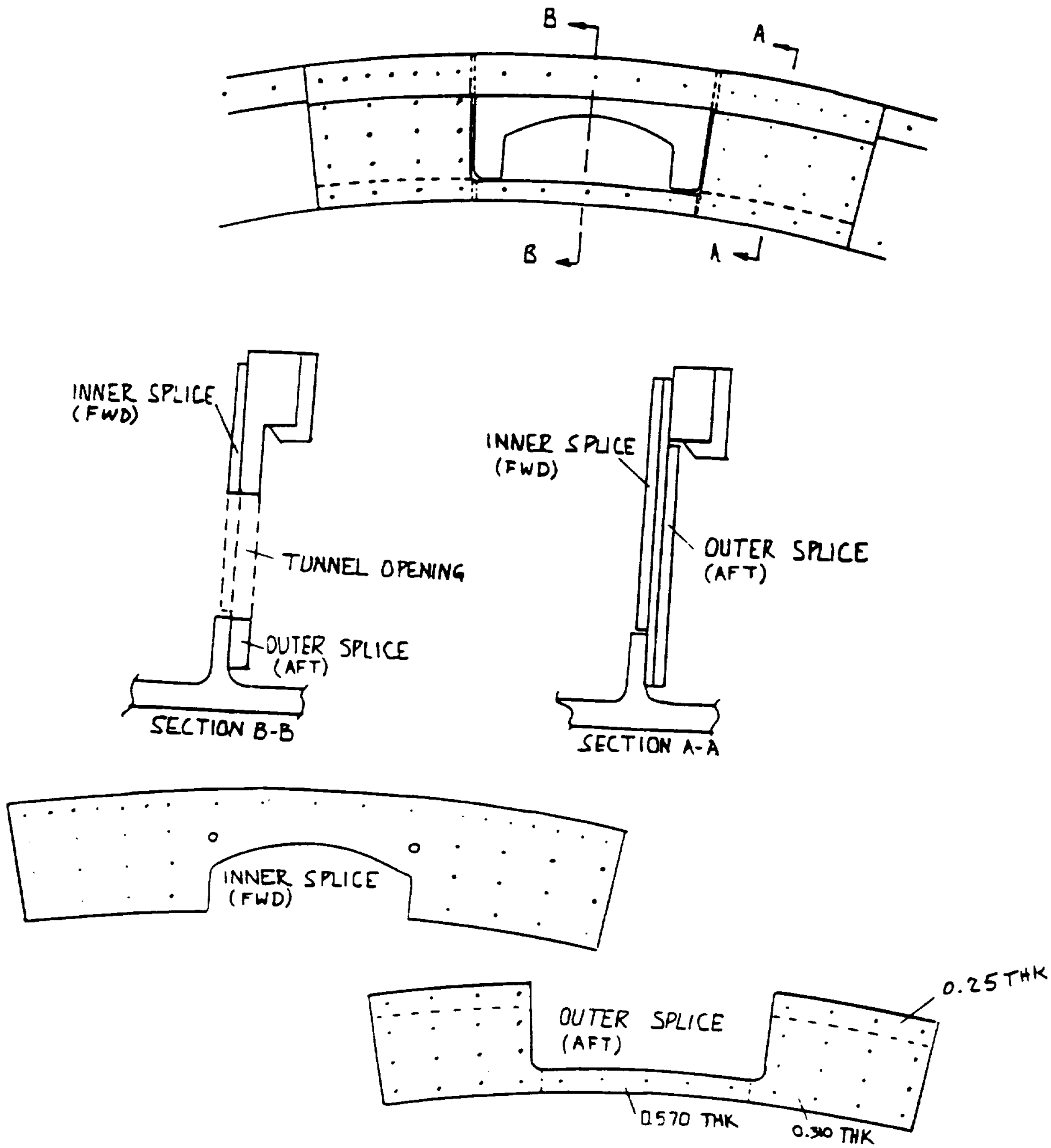


Figure 5. Systems tunnel splice (from ref. 1).

III. DETERMINISTIC FINITE ELEMENT ANALYSIS

Finite element analysis has been used[1] to analyze the ETA ring using the powerful finite element code NASTRAN. The finite element model is shown in figure 6 which actually shows the underformed plot of the ETA ring. It was developed using MSFC/NASTRAN, version 65A on the TBE VAX 11/780 computer system and executed on the Cray system at MSFC.

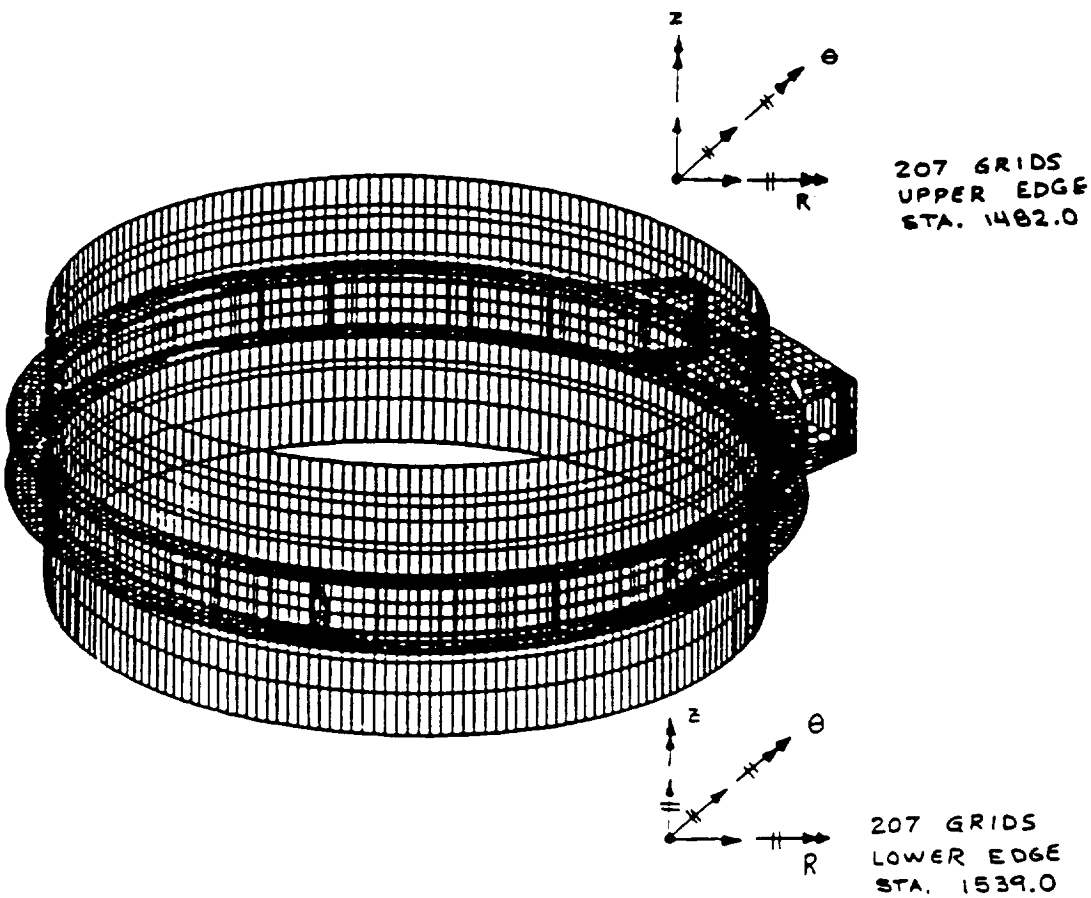


Figure 6. ETA ring NASTRAN model boundary conditions (from ref. 1).

The ETA ring finite element model is described in detail in references 1 and 2. The model consisted of approximately 10,000 nodes and 20,000 elements including some of the elements which are quadrilateral and bar elements. The ETA ring modeling details are shown in figure 7. The boundary conditions imposed on the NASTRAN model are shown in figure 6. NASTRAN output was obtained in the form of a punch file for the 18 "unit" load cases described below.

1. SRM radial internal pressure (100 lb/in^2)

2. Upper strut aligned (1 kip tension)

3. Upper strut aligned (1 kip compression)

4. Upper strut misaligned 90° forward (1 kip)

5. Upper strut misaligned 90° aft (1 kip)

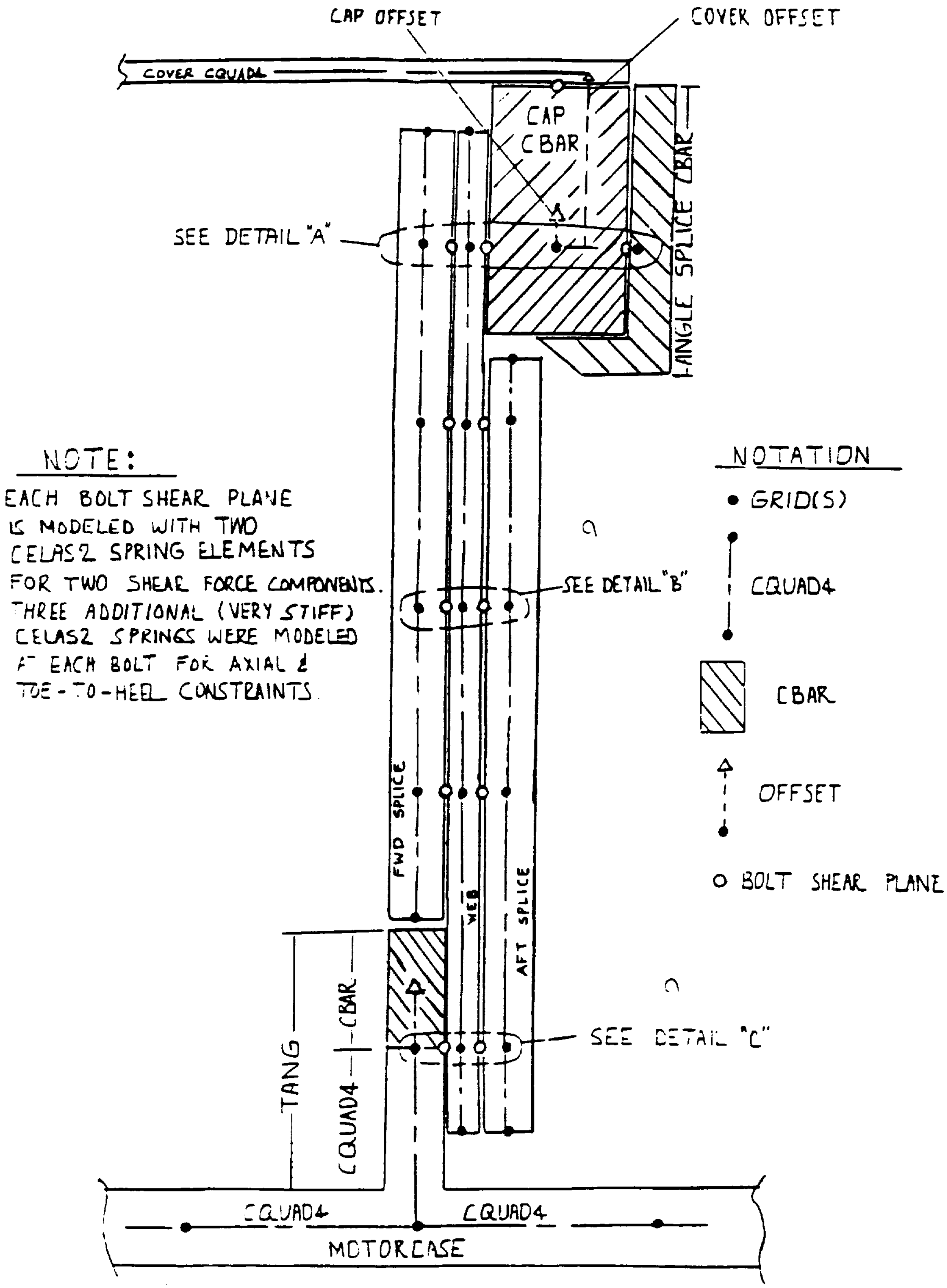


Figure 7. ETA ring modeling details (from ref. 1).

6. Lower strut aligned (1 kip tension)

7. Lower strut aligned (1 kip compression)

8. Lower strut misaligned 90° forward (1 kip)

9. Lower strut misaligned 90° aft (1 kip)

10. Diagonal strut aligned (1 kip tension)

11. Diagonal strut aligned (1 kip compression)

12. Diagonal strut misaligned 90° forward (1 kip)

13. Diagonal strut misaligned 90° aft (1 kip)

14. SRM axial load (10^6 lb)

15. SRM bending moment M_y (10^6 in-lb as sinusoidal axial load)

16. SRM bending moment M_z (10^6 in-lb as sinusoidal axial load)

17. Web/IEA cover axial pressure (1 lb/in^2)

18. IEA cover radial pressure (1 lb/in^2).

FORTRAN postprocessing codes were written to multiply the "unit" results by the actual loads and linearly combine the results to create stresses and forces for each element and each load case. As pointed out earlier, the critical sections to be studied in the probabilistic analyses (based on the deterministic finite element analyses) are tunnel splices and H-fitting. The load conditions studied are: USLI108A, USLI045A, USHI005D, and USHI006D. For example, for USLI108A, the corresponding equation for the total stress or force R_T (for 0° alignment) is given by:

$$R_T = 8.55R_1+9.15R_4+0.0R_{17}+0.0R_{18}+60.5R_{15}+44.0R_{16}+177.1R_3+49.5R_6+17.6R_{11} \; . \tag{1}$$

Similarly, the required forces can be obtained for other load conditions using the analysis load combination in table 1 for the above-mentioned cases. For all combinations, the critical loads were found to be lift off and high-Q load conditions.[1] Figure 1 and table 1 give details of these conditions. As will be shown later, this method is intended to be used as one of the alternate approaches to simulate the load conditions for finite element analysis and the corresponding principal stresses without actually rerunning the finite model of the ETA ring.

Using the standard techniques of theory of elasticity, the stresses were calculated deterministically for each ring component at critical sections identified by the NASTRAN output postprocessing.

Net (minimum) section properties were used in these analyses based on the worst-case dimensions. This is the traditional minimum resistance-maximum load approach of deterministic analysis. The corresponding margins of safety (MS) are also calculated.

Table 1. Analyses load combinations.

Load Case ID	Int. Press (lb/in^2 absolute)	Axial Load (RO^b 1b)	Web Press (lb/in^2 gauge)	Cover Press (lb/in^2 gauge)	M_Y (10^6 lb in)	M_Z (10^6 lb in)	Upper Strut Load (kips)	Lower Strut Load (kips)	Diagonal Strut Load (kips)
<u>Lift-Off</u>									
USLI045A, 0°	855.0	9.15	0.0	0.0	60.5	44.0	55.0	204.6	–171.6
USLI108A, 0°	855.0	9.15	0.0	0.0	60.5	44.0	–177.1	49.5	–17.6
<u>High Q</u>									
USHL005D, 0°	802.0	7.27	–17.2	1.9	–73.7	–5.5	97.9	–62.7	223.3
USHL006D, 0°	664.0	7.27	–17.2	1.9	–73.7	–5.5	–107.8	174.9	–315.7

It is to be noted that all these calculations corresponding to the deterministic model are performed based on the assumption that the material and the load properties are also deterministic. Since in reality all variables have certain variation, small or large depending on the actual case, these should be considered in the analysis. Hence probabilistic analysis is performed to study the actual safety reserve.

IV. BRIEF LITERATURE REVIEW

There has been a significant amount of work done in the past two decades in the area of reliability. Notable of these are references 3 to 9 in terms of the basic development of reliability concepts. In addition, there have been several applications by various authors.[10–17] This report does not deal in detail with the work done by the above-referenced authors, but the work done for this report is a continuation along the same lines but applied to shuttle structure and specifically to the ETA ring. It should also be pointed out at this stage that the literature review quoted here relates to the work done for the present report and is by no means an exhaustive review of all the work done in the reliability area.

V. RELIABILITY ANALYSIS

To perform a reliability analysis, the formulation of limit state function g is essential. The corresponding safety index (β) which is defined in subsequent paragraphs can then be calculated.

In this study, the reliability index (β) for a ring connecting the SRB to the ET for the limit state of stress is to be calculated. β in turn depends on the mean value standard deviation of the random variables connected with the material, geometric, and load properties. Specifically, β can be defined as

$$\beta = \frac{\bar{g}}{\sigma_g} , \tag{2}$$

where g is the limit state function, $\bar{g}$ is the mean value, and σ_g is the standard deviation of g. g in turn can be expressed as

$$g = R-S , \tag{3}$$

where R is the resistance and S is the load function for the structure under consideration. As can be seen in figure 8, β can be considered as the measure of the number of standard deviations that the mean value of the limit state function is from the failure surface. Figure 9 shows the demarcation of failure and survival states.[14] If R and S both are normal or lognormal, β and the corresponding probability of failure are related as

$$\beta = \Phi^{-1}(1-P_f) , \tag{4a}$$

or simply

$$p_f = \Phi(-\beta) , \tag{4b}$$

where Φ is the standard cumulative normal distribution function, and Φ^{-1} is the inverse of the standard cumulative normal distribution function. Figure 10 shows the actual p_f when R and S are both random variables. The reliability R can then be expressed as

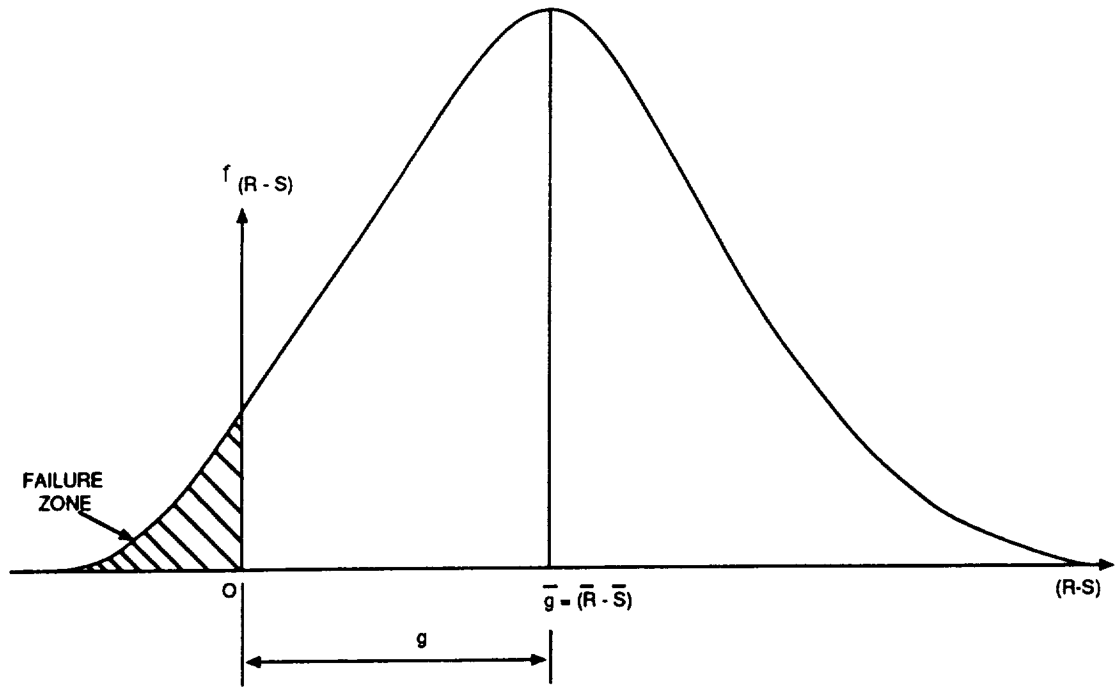


Figure 8. Definition of β and density function of limit state.

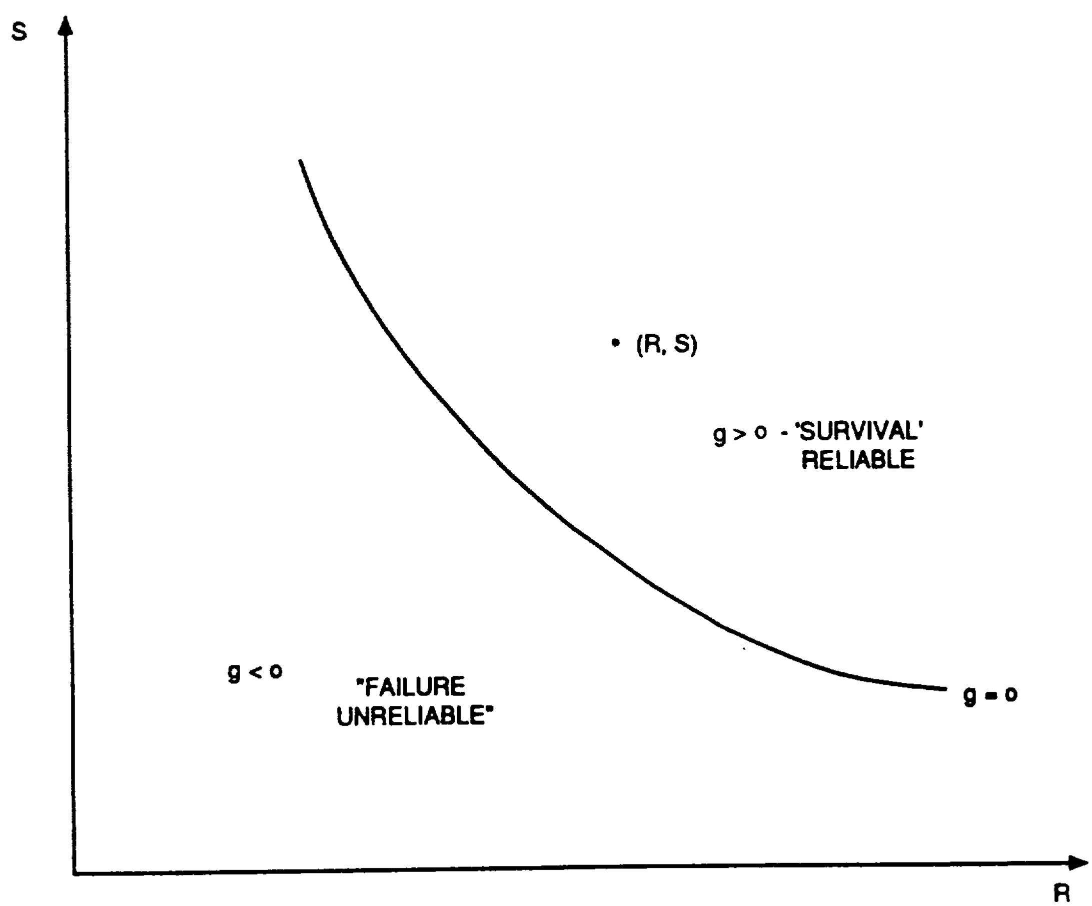


Figure 9. Conceptualization of limit state.

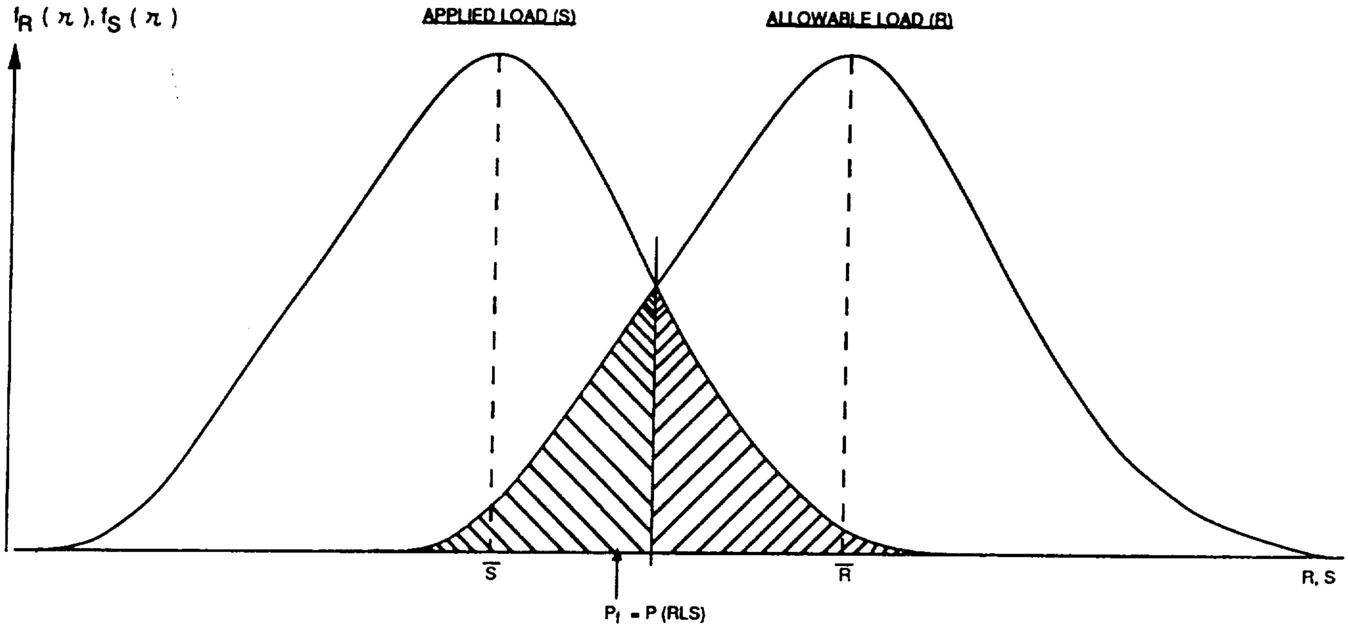


Figure 10. Probability of failure measure.

$$R = 1 - p_f . \tag{5}$$

This implies that an increase in β leads to an overall increase in the reliability of the structure. It should be noted that equations (4*a*) and (4*b*) are not exact when the resistance and the load parameters are not normally distributed and/or when the limit state function is not linear in its variables.

The problem would be simple if there exists a closed-form expression for the resistance variable, load variable, and the limit state function. Even in those cases, the exact evaluation of the probability of failure integral is almost impossible for non-normal cases. The general probability of failure function is given:

$$p_f = \int \cdots \int f_x(x_1, x_2, \ldots, x_n)\, dx_1 dx_2 \ldots dx_n , \tag{6}$$

in which f_x is the joint probability density function for $x_1, x_2, \ldots, x_n$ and the integration is performed over the region where $g < 0$. The above expression is based on a limit state function of $g(x_1, x_2, \ldots, x_n) = 0$. In such cases, Monte Carlo simulation techniques are used to evaluate p_f, R, or β.

In the present study, as the structure is complicated there are no closed-form relations between the output variable of stress and the corresponding input variables. Hence, probabilistic study is even more complicated in this case. The deterministic procedure used for the calculation of stresses is the well-known finite element method (FEM). The corresponding probabilistic study, which will use Monte Carlo techniques, will involve an FEM method in conjunction with the properties of the probability distributions of all variables involved in the calculation of the stresses at critical points.

Two approaches were considered in this stage. The first approach was to treat the material and load properties as random variables while treating the geometric properties as deterministic. In this case, actual finite element analysis is to be performed for simulated load and material properties to give principal stresses using Monte Carlo simulation (discussed in subsequent paragraphs). The geometric properties were to be considered as deterministic to avoid remodeling of this complex mesh, realizing that this is a shortcoming of this approach. The second approach is to treat the material, geometric, and load properties as random variables but utilizing the results of the deterministic finite element analysis. This is done by considering the variation of cross-sectional parameters, variation of grid point forces, membrane forces, and bending moments, etc. (output of deterministic finite element analysis), and variation of allowable stress of steel into the model and using Monte Carlo simulation to get the simulated principal stresses. It is to be noted that in this approach the load and material properties are not explicit random variables, but implicitly they are treated as random variables by treating the grid point forces and membrane forces, etc., as random variables. It is this second approach that is mainly used in this report.

The procedure is explained with respect to the limit state of stress. For a general case, the probability of failure for the limit state of stress can be expressed as:

$$p_f = p(\sigma_{ACT} > \sigma_{ALL}) \ , \tag{7a}$$

or

$$p_f = p(\sigma_{ALL} < \sigma_{ACT}) \ . \tag{7b}$$

This is like $p_f = p(R < S)$ as traditionally used, where R is the combined resistance function, and S is the combined load function. The word combined is used here because σ, whether it is actual or allowable, is based on the material, geometric, cross-sectional, and load properties, and hence, σ_{ALL} and σ_{ACT} are not clear cut resistance (where, in general, resistance function would involve only material, geometric, and cross-sectional properties) and load (would involve only load parameters). σ_{ACT} can be expressed as:

$$\sigma_{ACT} = f_1(x_{m1}, x_{m2}, \ldots, x_{mn}, x_{g1}, x_{g2}, \ldots, x_{gn}, x_{c1}, x_{c2}, \ldots, x_{cn}, x_{L1}, x_{L2}, \ldots, x_{Ln}) \ , \tag{8}$$

where

x_{m1} = first variable representing material property

x_{g1} = first variable representing geometric property

x_{c1} = first variable representing cross-sectional property

x_{L1} = first variable representing load properties directly or the output parameter of the finite element analyses (like the membrane force, grid point force, etc.).

In the present study, σ_{ACT} is the principal stress σ_P. The design nominal (deterministic) values of these variables were obtained from the structures materials/load group[1] at MSFC and from the available literature. Since a probabilistic study is to be performed with the intent of final calculation of reliability, the variation of these parameters is required. To be specific, the statistical properties (like the mean value and coefficient of variation) of the distribution and the type of the distribution followed by these variables is required. The relevant expressions are given as:

$$\bar{X} = \bar{B}\hat{\bar{X}} \ , \tag{9}$$

$$V_x = \left[\hat{V}_x^2 - \hat{V}_B^2\right]^{1/2} \ , \tag{10}$$

$$V_B = \left[V_1^2 + V_2^2 + \ldots V_n^2\right]^{1/2} \ , \tag{11}$$

where $\hat{\bar{X}}$ and $\hat{V}_x$ are the estimates of the mean value and the coefficient of variation based on the available data; $\bar{X}$ and V_x are the "true" mean and coefficient of variation of the variable; and $\bar{B}$ and V_B are the mean value and standard deviation of bias factor. If the model is unbiased, $\bar{B} = 1$ and $V_B = 0$. If the data are limited, $\hat{\bar{x}}$ and $\hat{V}_x$ can be calculated from,

$$\hat{\bar{x}} = \frac{x_1 + x_2}{2} \ , \tag{12}$$

$$\hat{V}_x = \frac{x_2 - x_1}{\frac{1}{2}(x_2 + x_1)} \ , \tag{13}$$

where it is assumed that 95 percent of the values lie between x_1 and x_2. x_1 and x_2 can be considered as the practical extreme limits.

Similarly, σ_{ALL} can be expressed as,

$$\sigma_{\text{ALL}} = f_2(x_{m1}, x_{m2}, \ldots, x_{mn}, x_{g1}, x_{g2}, \ldots, x_{gn}, x_{c1}, x_{c2}, \ldots, x_{cn}, x_{L1}, x_{L2}, \ldots, x_{Ln}) \ , \tag{14}$$

which is a different function using the allowable values of the input variables. Alternatively, σ_{ALL} may be deterministic if it is a value taken from the governing codes for that structure. In any case, there is no closed-form expression for σ_{ACT} or σ_{ALL}. The computer will take the random values of the input variables, go through finite element code (or use the results with a certain coefficient of variation), and using Monte Carlo techniques will calculate the stresses at different critical points. Once finite element analysis of these hypothetical rings is done (or the variation of the output parameters is considered) in conjunction with Monte Carlo techniques, a statistical analysis is done to calculate p_f and the safety index along with the reliability level using equations discussed above. Similar reliability analysis can be done for other limit states of the structure. In the present study, σ_{ALL} is $(\sigma_u)_{\text{ALL}}$ the ultimate allowable or (σ_y) the yield allowable stress depending on the actual case being considered.

The detailed procedure of Monte Carlo simulation in conjunction with the deterministic finite element analysis is given below:[18]

1. Generate a random number r_i between 0 and 1 using Rand Corporation procedure or standard subroutines in the computer corresponding to a uniform distribution for any input variable dealing with cross section, load/force parameter and also for allowable stress. Note at this stage all the necessary random numbers for all input variables are generated either dealing with both actual stress σ_{ACT} or σ_{p} and also allowable stress for σ_{ALL}.

2. For the actual random value to be generated for any variable corresponding to a given distribution, the statistical properties of that variable are needed, like $\hat{\bar{x}}$ and V_x. Calculate $\hat{\bar{x}}$ and V_x from the available experimental data and then calculate $\bar{x}$ and V_x from equations (9) to (11) using the proper bias

factor. In the absence of sufficient data for any of the input random variable x, calculate $\hat{\bar{x}}$ and V_x from equations (12) and (13).

3. Since all the variables are assumed to be normal, the transformation of random numbers from uniform to normal is performed using,

$$C_i = \Phi^{-1}(r_i) \ , \tag{15}$$

where C_i is a standardized random number such that it is normally distributed with mean = 0 and variance = 1. Φ^{-1} is the inverse of the standard cumulative normal distribution function. Note that standard subroutines are available to perform such an operation. IT is also to be noted that equation (15) uses the r_i values generated in step 1.

4. Knowing C_i values from equation (15) and $\bar{x}$ and V_x from step 2, a random value of x can then be generated using

$$x_i = \bar{x} + s_x C_i \ , \tag{16}$$

where s_x is the standard deviation of x given by $s_x = \bar{x}V_x$.

5. Generate random values for all independent variables (as mentioned in step 1) following steps 1 to 4 and then using Monte Carlo simulation and the equations of theory or elasticity to calculate principal stresses for all the critical elements of these hypothetical rings subjected to hypothetical loads/forces. It is to be noted that the word hypothetical is not used to indicate that these rings and the loads are all fictitious but instead it indicates that the deterministic ring problem is now studied by perturbing its material, geometric, and load properties consistent with the actual structural behavior. Since it is now widely accepted that no variable is truly deterministic, this study attains significant importance.

6. Having obtained a set of random values for each of the dependent variables like σ_{ALL} and σ_{ACT}, perform statistical analysis of the required parameters. This could be the mean value, standard deviation, and coefficient of variation of all the response parameters.

7. Calculate the probability of failure of an event corresponding to equation (7*b*) from

$$p_f = \frac{n_1}{n} \ , \tag{17}$$

where n_1 is the number of times that an event has occurred in a sample size of n. For example, to calculate p_f corresponding to equations (7*a*) and (7*b*), once a random number has been generated for σ_{ALL} or σ_{p} and σ_{ACT}, it is checked if $\sigma_{\mathrm{ALL}} < \sigma_{\mathrm{ACT}}$. If it is true, then it corresponds to a failure event. Such occurrences are totaled for the sample size, and p_f is then calculated from equation (17).

8. Knowing p_f, one can calculate reliability R from equation (5).

9. The safety index β can be calculated from equation (4*a*) if the distribution of all the input variables is normal. Otherwise β can be calculated using equation (2) where g and σ_g are the results obtained from step 6. However, this approach is not used in this study as all variables are assumed normal.

10. Steps 6 to 9 can be repeated for all the limit states of the ring problem. In the present study, only one limit state (stress) is considered.

11. The effect of coefficient of variation of some of the critical input parameters on reliability of the ring (with respect to various limit states) will be studied as part of the overall behavioral study of the ring.

If resistance and load functions are approximated as normal, then the safety index (β) has a closed-form expression given by,

$$\beta = \frac{\bar{R}-\bar{S}}{\sqrt{\sigma_R^2+\sigma_S^2}} = \frac{(n-1)}{\sqrt{n^2V_R^2+V_S^2}} \;, \tag{18}$$

where $n = \bar{R}/\bar{S}$, $V_R = \sigma_R/\bar{R}$, and $V_S = \sigma_S/\bar{S}$. For the notations of this report, R is σ_{ALL} and S is the σ_{ACT} or σ_p. Similarly for resistance and load functions being lognormal, β is given by,

$$\beta = \frac{\ln(n)}{\sqrt{V_R^2+V_S^2}} \;. \tag{19}$$

For the variables not conforming to the above standard cases, β can be obtained numerically. As explained earlier, R and S in this report correspond to σ_{ALL} and σ_{ACT}, respectively.

VI. FORMULATION OF THE PROBLEM

As mentioned previously in this report, the components of the ETA ring that are critical were tunnel splices and H-fittings. In each of these components, there are few sections that were found critical based on deterministic analysis.[1] For all these cases, the reliability levels with respect to the limit state of stress are calculated. The final output of the probabilistic analysis is principal stress σ_p. Hence, the probability of failure corresponding to equation (7*b*) can be written as:

$$p_{fy} = p((\sigma_y)_{ALL} < \sigma_p) \;, \tag{20}$$

similarly for ultimate allowable stress

$$p_{fu} = p((\sigma_u)_{ALL} < \sigma_p) \;. \tag{21}$$

Knowing p_{fy} and p_{fu}, the corresponding reliability values R_y and R_u can be easily calculated from the generic equation (5) given earlier. The safety indices β can be calculated from equation (18) as the variables have been assumed to be normal.

At this point, it is important to note that in a deterministic analysis the MS is calculated using the equation given below:[1]

$$(MS)_{YIELD} = \frac{(\sigma_y)_{ALLOW}}{1.1\,(\sigma_p)_{ACTUAL}} - 1 \;, \tag{22}$$

for ultimate allowable stress:

$$(MS)_{\mathrm{ULT}} = \frac{(\sigma_U)_{\mathrm{ALLOW}}}{1.4\,(\sigma_p)_{\mathrm{ACTUAL}}} - 1\ . \tag{23}$$

The factors 1.1 and 1.4 that are used in equations (22) and (23) represent the safety factor in yield and ultimate allowable stress. Since safety factor is nothing but an ignorance factor and was introduced to compensate for all uncertainties, it should be discarded in reliability analysis as is traditionally done. That raises an important question with regard to equations (20) and (21). If in an analysis the data show that the basic variability in $(\sigma_y)_{\mathrm{ALL}}$ and $(\sigma_u)_{\mathrm{ALL}}$ is the same, that would indicate that p_{fu} is smaller than p_{fy} and hence is not as critical as the yield case. This is opposite to the general notion in deterministic analysis wherein the ultimate case is more critical than yield due to different safety factors. This should also be considered along with the fact that taking $sf = s/S$ (where s is the allowable stress and S is the actual stress). It can be proved that

$$V_{sf} = \sqrt{\sigma_s^2 + \sigma_S^2}\ ,$$

considering both as random variables. In cases where either σ_S or σ_s, the standard deviation in allowable and actual stress, is high, V_{sf}, the coefficient of variation could be high. In some cases, this may have to be included in the model as an additional uncertainty in addition to the usual uncertainty in material geometric and load properties making sure it is not considered twice. On the whole, the authors of this report feel that so far as the reliability analysis is concerned it is the ultimate allowable stress (limiting stress) that is important. Provided that data are collected properly and proper statistical tools are used, the uncertainty in material, geometric, and load properties should take care of the safety factor automatically, and the reliability analysis should give a reasonable idea of the safety reserve in the structure. The results in this report are given using limit state stress of $(\sigma_u)_{\mathrm{ALL}}$.

For a ready comparison of probabilistic margin of safety (PMS), values are calculated for comparison with the corresponding deterministic MS. This is given as

$$PMS = \frac{(\sigma_u)_{\mathrm{ALL}} - (\sigma_p)_{\mathrm{MAX}}}{S_{\sigma u}}\ , \tag{24}$$

where,

$$(\sigma_p)_{\mathrm{MAX}} = \bar{\sigma}_p + \beta S_{\sigma p}\ , \tag{25}$$

here,

$(\sigma_u)_{\mathrm{ALL}}$ = expected value of allowable stress

$S_{\sigma u}$ = standard deviation of allowable stress

$S_{\sigma p}$ = standard deviation of principal stress

$\bar{\sigma}_p$ = mean value of principal stress

β = safety index of the structure with respect to stress limit state.

The reliability analysis of the above mentioned critical components will now be discussed in detail.

The results for the basic cases of critical sections (cases A to E) are obtained for a sample of size of 5,000 random numbers using the variance reduction techniques[4] and for the following basic values of input parameters: $V_c = 0.01$, $V_f = 0.06$, $V_{AS} = 0.022$, $C_{MNC} = 1.15$, $C_{MNF} = 1.0$, and $C_{MNS} = 1.055$, where V stands for coefficient of variation, and C stands for coefficient of mean/normal. The subscripts C, F, and A_S for V stands for cross-sectional parameters, forces (grid point forces, membrane forces, and bending moments), and allowable stress, respectively. The subscripts MNC, MNF, and MNS for C stand for mean to nominal for cross-sectional parameters, mean to nominal for forces (grid point, membrane, and bending moments), and mean to nominal for allowable stress parameters, respectively. The effects of other V and C values are studied under sensitivity analysis. The V_c values are actually lower/higher than 0.01 for some cross-sectional parameters based on the general data in the literature.[12 16 20] To avoid any ambiguity, the probabilistic data for input parameters are tabulated for cases A to E when those cases are dealt with in the report individually.

A. 90° Forward Tunnel Splice Plate (Minimum Section)

This corresponds to USBI drawing 10170–0367 from reference 1. The general expression for principal stress is given by,[1]

$$\sigma_{p1},\sigma_{p2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}\ , \tag{26}$$

$$\sigma_p = \mathrm{MAX}(\sigma_{p1},\sigma_{p2})\ , \tag{27}$$

which is considered for reliability calculation.

$$\tau_{xy} = \frac{F_{xy}}{A_x}\ , \tag{28}$$

$$\sigma_y = \frac{F_y}{A_y} + \frac{M_y C_z}{I_{xc}}\ , \tag{29}$$

$$\sigma_x = \frac{F_x}{A_x} + \frac{M_z C_y}{I_z} + \frac{M_x C_z}{I_y}\ . \tag{30}$$

σ_x, σ_y, and τ_{xy} represent normal stresses in x and y direction and shearing stress, respectively. F_x, F_y, F_{xy}, M_x, M_y, and M_z are the element membrane forces and bending moments. A_x, A_y, I_{xc}, I_y, and I_z are cross-sectional properties. These parameters can be obtained knowing the critical cross section. The critical cross section for this is shown in figures 11 and 12.

It is to be noted that all the values shown are nominal/design values. Table 2 shows the list of all the basic input variables used in the probabilistic analysis including that for allowable stress of steel. The information regarding $\bar{x}/xn$ for any variable and the coefficient of variation is obtained from discussions with the relevant groups at MSFC and based on existing data in the literature.[14 20–22] The results are

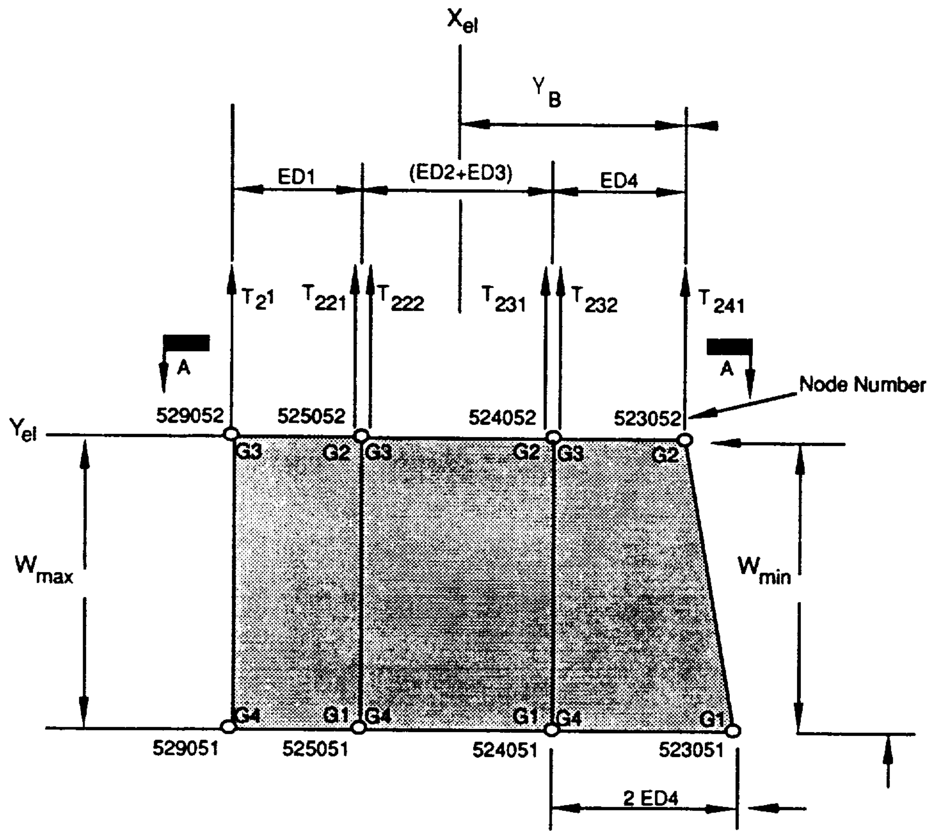


Figure 11. Cross-sectional parameters and grid point forces for critical section of case A (from ref. 1).

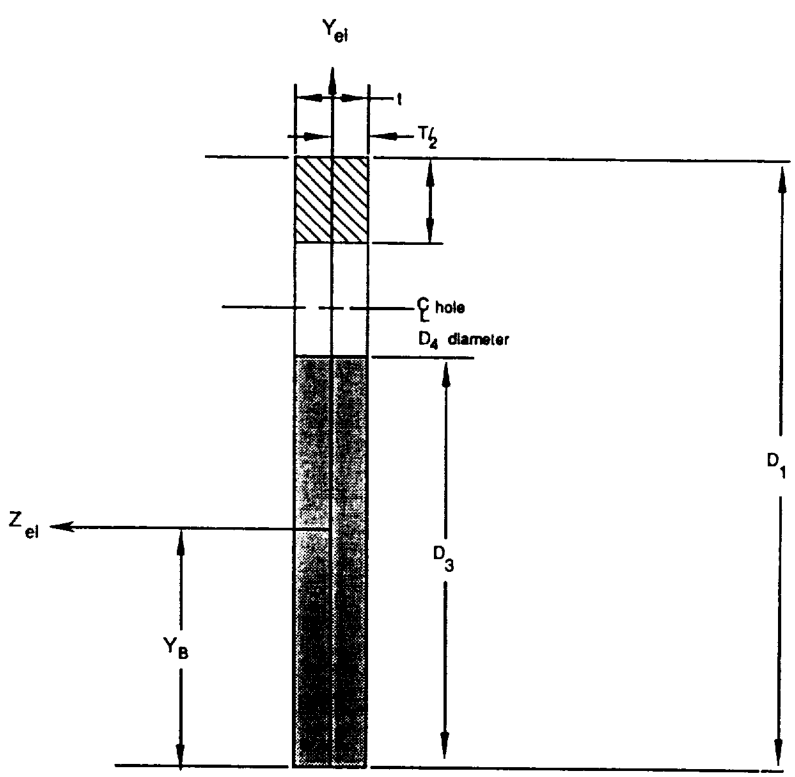


Figure 12. Cross-sectional parameters for critical section of case A (ref. 1) (view AA of fig. 11).

Table 2. Case A: Probabilistic input parameters.

Parameters	Nominal	Mean/Nom	Est. Cov.	Bias	Cov. Bias
T	0.240	1.15	0.0400	1.01	0.05
D1	2.690	1.15	0.0042	1.01	0.05
D2	0.285	1.15	0.0042	1.01	0.05
D4	0.510	1.15	0.0042	1.01	0.05
Wmin	2.369	1.15	0.0042	1.01	0.05
Wmax	2.452	1.15	0.0042	1.01	0.05
T21	2,105.000	1.0	0.060	0.95	0.05
T221	4,147.000	1.0	0.060	0.95	0.05
T222	6,431.000	1.0	0.060	0.95	0.05
T231	10,178.000	1.0	0.060	0.95	0.05
T232	8,157.000	1.0	0.060	0.95	0.05
T241	11,615.000	1.0	0.060	0.95	0.05
Fy	1,079.000	1.0	0.060	0.95	0.05
Mx1	–2.160	1.0	0.060	0.95	0.05
Mx2	32.300	1.0	0.060	0.95	0.05
Mx3	57.490	1.0	0.060	0.95	0.05
My	69.220	1.0	0.060	0.95	0.05
Fxy1	–590.500	1.0	0.060	0.95	0.05
Fxy2	–961.400	1.0	0.060	0.95	0.05
Fxy3	–162.000	1.0	0.060	0.95	0.05
FSY	1.100	1.0	0.01	1.00	0.00
FSU	1.400	1.0	0.01	1.00	0.00
SIGy	163.000	1.055	0.022	1.00	0.00
SIGu	180.000	1.055	0.022	1.00	0.00

obtained in terms of probability of failure using Monte Carlo simulation discussed earlier in this report. The corresponding reliability is then calculated from equation (5). The safety index β and PSM are calculated from equations (18) and (24), respectively. The results are shown below:

$\bar{\sigma}_p = 97.98$ $\quad S_{\sigma p} = 9.39$ $\quad \bar{\sigma}_u = 189.9$ $\quad S_{\sigma u} = 4.17$

$p_{fu} = 0.15\times10^{-14}$ $\quad R_u = 0.9_{(14)}$ $\quad \beta = 8.93$ $\quad PMS = 1.89$.

This shows a high PMS compared to the corresponding deterministic MS of –0.03. It is to be noted that the principal stress has a high coefficient of variation. Figure 13 shows the Warner diagram for case A. A detailed discussion appears later.

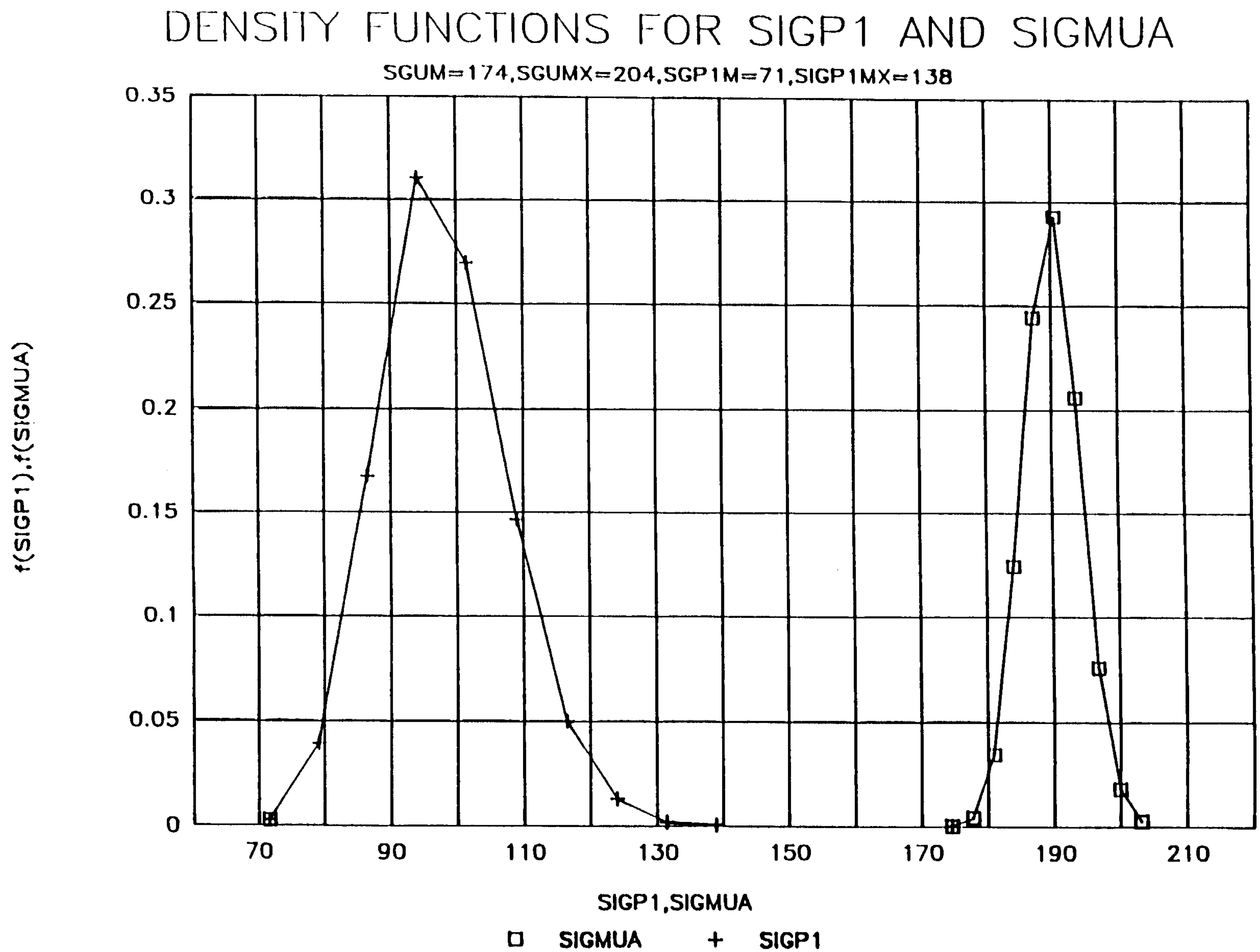


Figure 13. Warner diagram for case A.

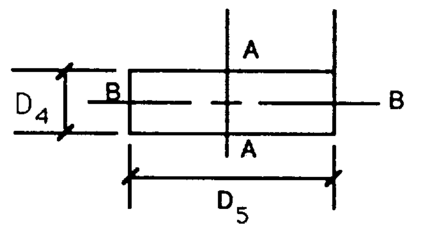
B. 90° Aft Tunnel Splice (Small Section)

This corresponds to USBI drawing 10170–0372 from reference 1. The critical section is shown in figure 14. The input data are shown in table 3. The results are shown below.

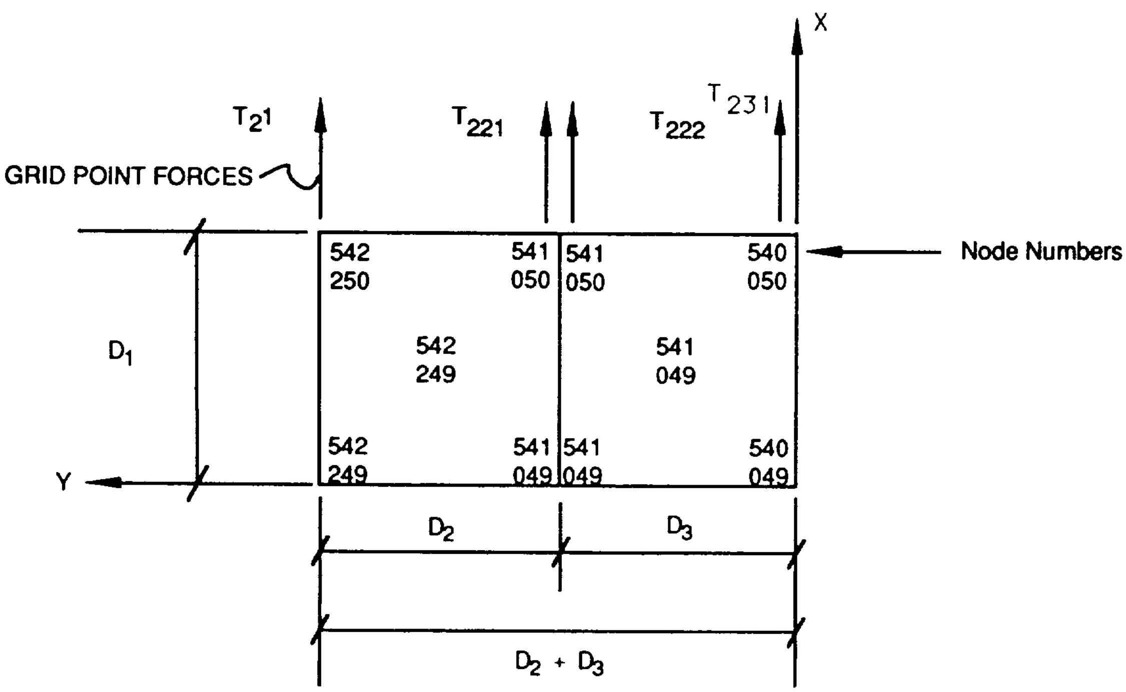
$\bar{\sigma}_p = 131.39$ $\quad S_{\sigma p} = 14.98$ $\quad \bar{\sigma}_u = 189.9$ $\quad S_{\sigma u} = 4.17$

$p_{fu} = 8.03\times10^{-4}$ $\quad R_u = 0.99919$ $\quad \beta_u = 3.7604$ $\quad PMS = 0.5143$.

Again a good reliability level is seen in the results. The PMS of 0.5143 is much higher than the corresponding deterministic MS of –0.24. The corresponding probability of failure diagram is shown in figure 15.



a) CROSS SECTIONAL PROPERTIES



b) CROSS SECTIONAL PROPERTIES AND GRID POINT FORCES

Figure 14. Cross-sectional properties and grid point forces for critical section of case B (from ref. 1).

Table 3. Case B: Probabilistic input parameters.

Parameters	Nominal	Mean/Nom	Est. Cov.	Bias	Cov. Bias
D1	2.250	1.15	0.0042	1.01	0.05
D2	0.590	1.15	0.0042	1.01	0.05
D3	0.600	1.15	0.0042	1.01	0.05
D4	0.300	1.15	0.0042	1.01	0.05
D5	1.160	1.15	0.0042	1.01	0.05
T21	10,967.000	1.0	0.06	0.95	0.05
T221	6,243.000	1.0	0.06	0.95	0.05
T222	10,236.000	1.0	0.06	0.95	0.05
T231	6,681.000	1.0	0.06	0.95	0.05
Fx1	29,176.000	1.0	0.06	0.95	0.05
Fx2	28,134.000	1.0	0.06	0.95	0.05
Mx1	485.000	1.0	0.06	0.95	0.05
Mx2	453.000	1.0	0.06	0.95	0.05
Fy	–917.000	1.0	0.06	0.95	0.05
My1	41.200	1.0	0.06	0.95	0.05
Fxy1	2,117.000	1.0	0.06	0.95	0.05
Fxy2	1,675.000	1.0	0.06	0.95	0.05
FSY	1.100	1.0	0.06	1.00	0.00
FSU	1.400	1.0	0.06	1.00	0.00
SIGy	163.000	1.055	0.022	1.00	0.00
SIGu	180.000	1.055	0.022	1.00	0.00

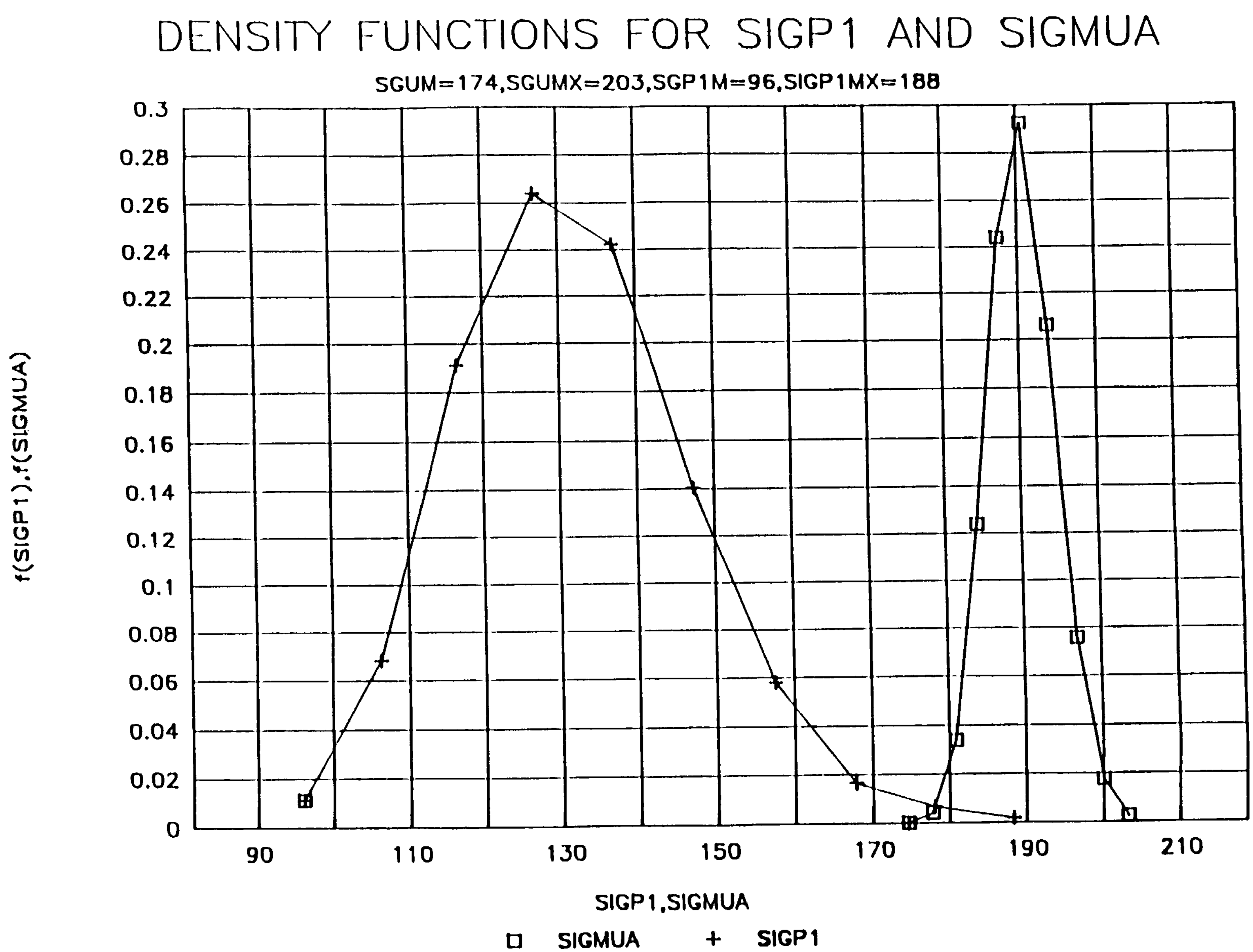


Figure 15. Warner diagram for case B.

C. 90° Forward Tunnel Splice (Critical Section)

This corresponds to USBI drawing 10170–0367 of reference 1. The critical sections are shown in figure 16. The probabilistic input parameters used for reliability analysis are shown in table 4. The results are shown below.

$\bar{\sigma}_p = 118.27$ $\quad S_{\sigma p} = 21.39$ $\quad \bar{\sigma}_u = 189.9$ $\quad S_{\sigma u} = 4.17$

$p_{fu} = 0.0036$ $\quad R_u = 0.9964$ $\quad \beta_u = 3.285$ $\quad PMS = 0.317$.

The corresponding deterministic MS is –0.22. This shows that probabilistic analysis has predicted a higher safety reserve than was calculated in the deterministic analysis. The corresponding Warner diagram is shown in figure 17.

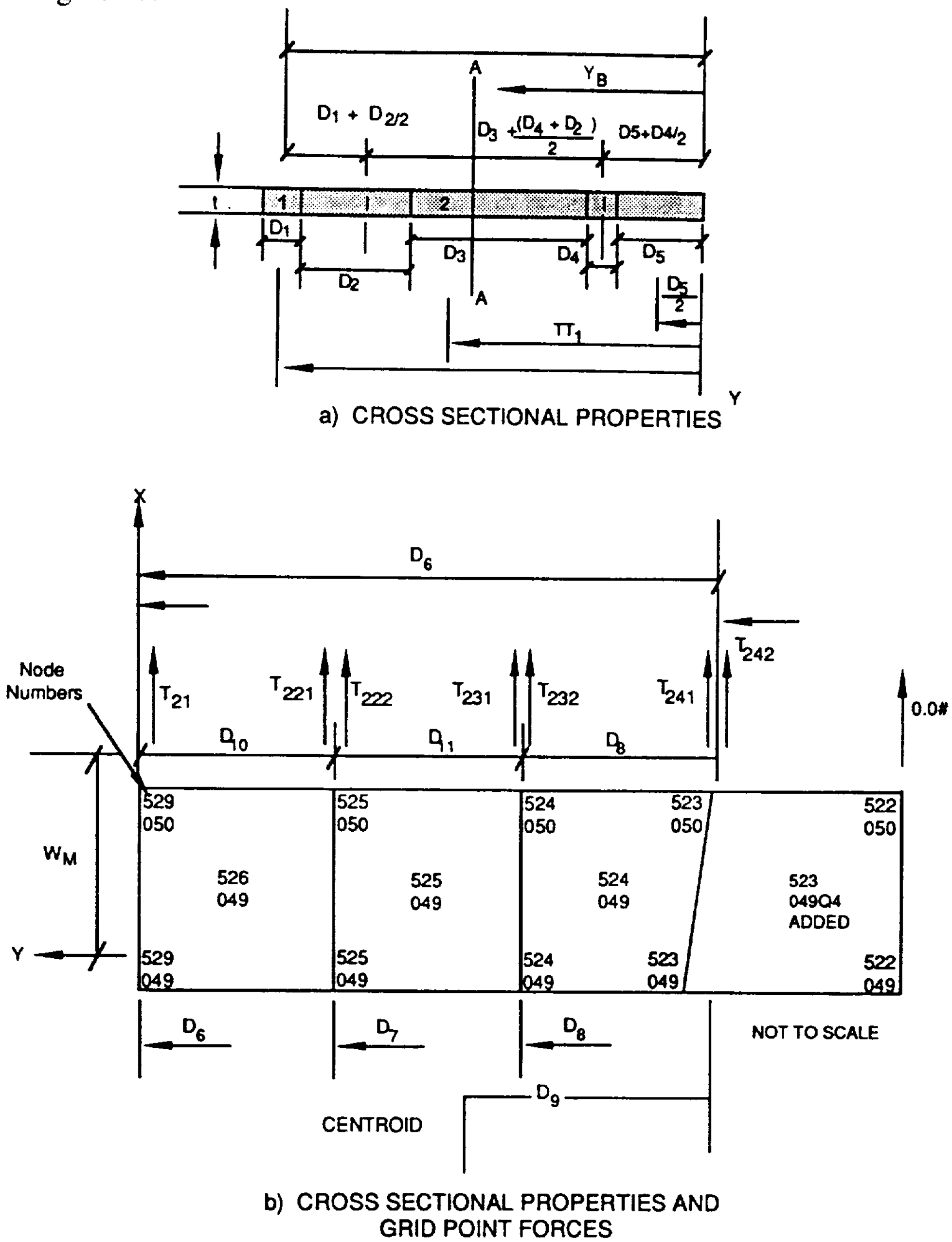


Figure 16. Cross-sectional properties and grid point forces for critical section of case C (from ref. 1).

Table 4. Case C: Probabilistic input parameters.

Parameters	Nominal	Mean/Nom	Est. Cov.	Bias	Cov. Bias
T	0.235	1.15	0.0400	1.01	0.05
D1	0.285	1.15	0.0042	1.01	0.05
D2	0.510	1.15	0.0042	1.01	0.05
D3	2.134	1.15	0.0042	1.01	0.05
D4	0.202	1.15	0.040	1.01	0.05
D5	0.769	1.15	0.040	1.01	0.05
D6	4.030	1.15	0.0042	1.01	0.05
D7	3.210	1.15	0.0042	1.01	0.05
D8	1.960	1.15	0.0042	1.01	0.05
D9	1.800	1.15	0.0042	1.01	0.05
D10	0.820	1.15	0.0042	1.01	0.05
D11	1.250	1.15	0.0042	1.01	0.05
D12	1.350	1.15	0.0042	1.01	0.05
D13	0.740	1.15	0.0042	1.01	0.05
D14	1.660	1.15	0.0042	1.01	0.05
D15	1.040	1.15	0.0042	1.01	0.05
Wm	2.450	1.15	0.0042	1.01	0.05
T21	7,498.000	1.0	0.060	0.95	0.05
T221	10,470.000	1.0	0.060	0.95	0.05
T222	10,749.000	1.0	0.060	0.95	0.05
T231	15,603.000	1.0	0.060	0.95	0.05
T232	13,299.000	1.0	0.060	0.95	0.05
T241	11,100.000	1.0	0.060	0.95	0.05
T242	5,717.000	1.0	0.060	0.95	0.05
Fy	1,077.000	1.0	0.060	0.95	0.05
Mx1	24.100	1.0	0.060	0.95	0.05
Mx2	75.600	1.0	0.060	0.95	0.05
Mx3	–8.900	1.0	0.060	0.95	0.05
Mx4	–6.700	1.0	0.060	0.95	0.05
My1	237.000	1.0	0.060	0.95	0.05
Fxy1	–1,005.000	1.0	0.060	0.95	0.05
Fxy2	–2,539.000	1.0	0.060	0.95	0.05
Fxy3	–399.200	1.0	0.060	0.95	0.05
Fxy4	402.000	1.0	0.060	0.95	0.05
Fx	5,476.000	1.0	0.060	0.95	0.05
Fsy	1.100	1.0	0.01	1.00	0.00
Fsu	1.400	1.0	0.01	1.00	0.00
SIGy	163.000	1.055	0.022	1.00	0.00
SIGu	180.000	1.055	0.022	1.00	0.00

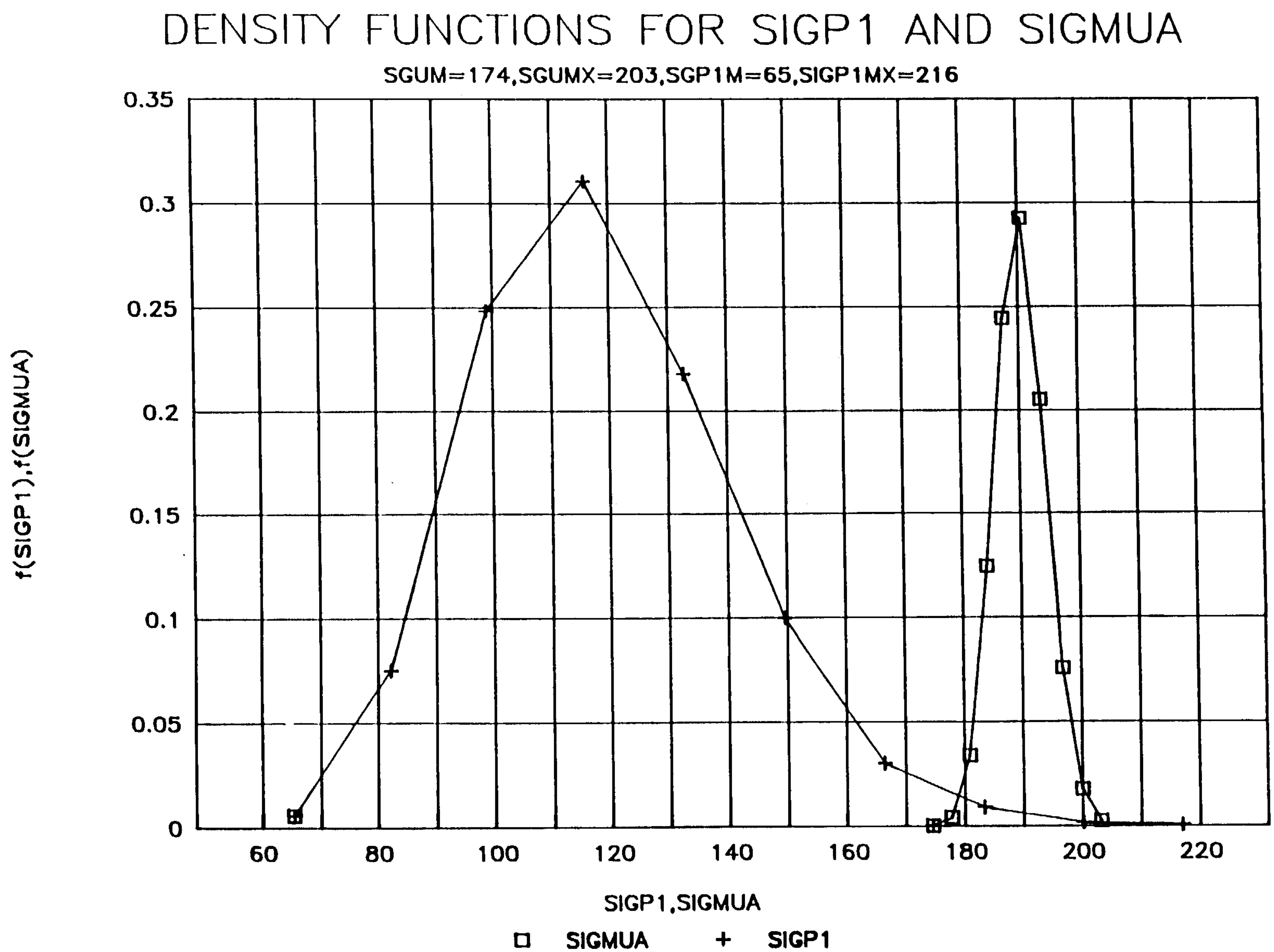


Figure 17. Warner diagram for case C.

D. H-Fitting (Plates)

This corresponds to USBI drawing 10170–0011 from reference 1. The critical section and the associated cross-section parameters are shown in figures 18 and 19. The probabilistic input parameters are shown in table 5. The results of reliability are shown below.

$\bar{\sigma}_p = 72.396$ $\quad S_{\sigma p} = 6.503$ $\quad \bar{\sigma}_u = 189.9$ $\quad S_{\sigma u} = 4.177$

$p_{fu} = (<10^{-14})$ $\quad R_u = 1.09$ $\quad \beta_u = 15.201$ $\quad PMS = 4.461$.

It is to be noted that the PMS is quite high. The corresponding deterministic MS was +0.52. The corresponding Warner diagram is shown in figure 20.

E. H-Fitting (Lugs)

This corresponds to USBI drawing 10170–011 of reference 1. The critical section and the associated cross-sectional parameters are shown in figures 21 and 22. The probabilistic input parameters are shown in table 6. The results are shown below.

$\bar{\sigma}_p = 116.15$ $\quad S_{\sigma p} = 14.39$ $\quad \bar{\sigma}_u = 189.9$ $\quad S_{\sigma u} = 4.17$

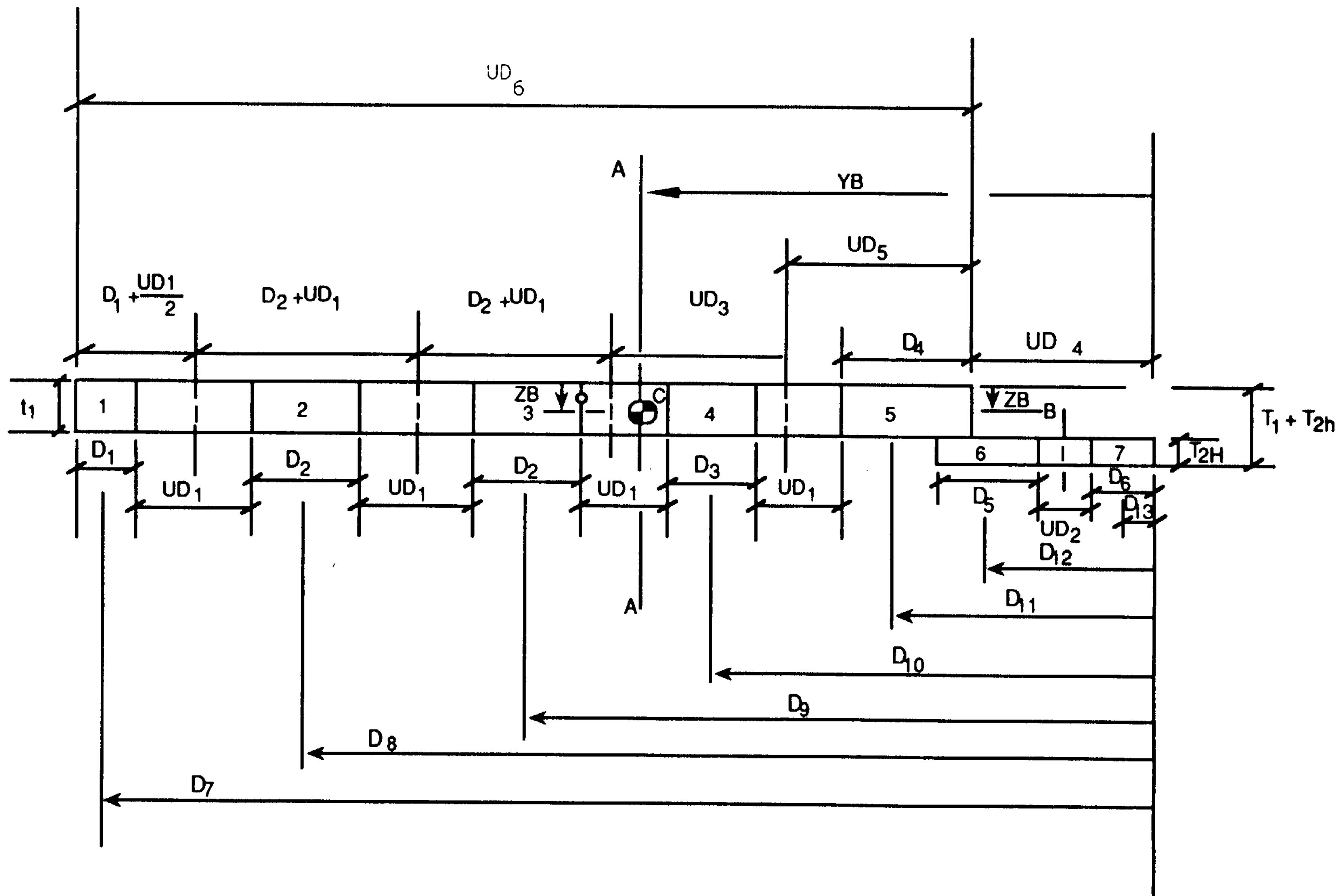
$p_{fu} = 0.37\times10^{-6}$ $\quad R_u = 0.9_{(6)}$ $\quad \beta_u = 4.920$ $\quad PMS = 0.699$.

Once again it has been shown that PMS (0.699) is much higher than the deterministic MS of –0.21. The corresponding Warner diagram is shown in figure 23.

VII. SENSITIVITY ANALYSIS

As mentioned above, cases A to E were studied for some basic core parameters as discussed in section VI. Since no actual experimental data were available for the input parameters and the data used were based on the available data in the literature and the discussions with the load and stress group at MSFC, it was found prudent to study the effect of variation of the input parameters on the reliability of the ETA ring for the stress limit state for most critical components at critical sections. Based on the results obtained for cases A to E and looking at the corresponding Warner diagrams, it is seen that the overlap between the actual principal stress and the allowable ultimate stress is higher for cases B and E compared to cases A, C, and D; while the PMS is consistently higher than the corresponding MS for deterministic analysis. Hence, the sensitivity analysis study is conducted for case B (tunnel splice plate) and case E (H-fitting lugs).

This study is done by studying the effect of variation of coefficient of cross-sectional parameters (V_c), force parameters (V_f) like membrane forces, bending moments, etc., allowable stress parameters (V_{AS}) on the reliability of the ETA ring, taken one at a time. The basic values considered are $V_c = 0.01$, $V_f = 0.06$, and $V_{AS} = 0.022$. These results are shown in figures 24 to 27 for case B and in figures 28 to 31 for case E. In addition to this, the effect of $\bar{x}/xn$ denoted as c_{mn} (coefficient of mean to nominal), on reliability is studied for various cross-sectional parameters (C_{mnc}), force parameters (C_{mnf}), and



UD1 = .379, UD2 = .521, UD3 = 0.80, UD4 = 1.440, UD5 = 1.555, UD6 = 5.16
(THESE DIMENSIONS ARE NOT USED EXPLICITLY IN CALCULATION).

Figure 18. Cross-sectional properties for the critical section of case D (from ref.1).

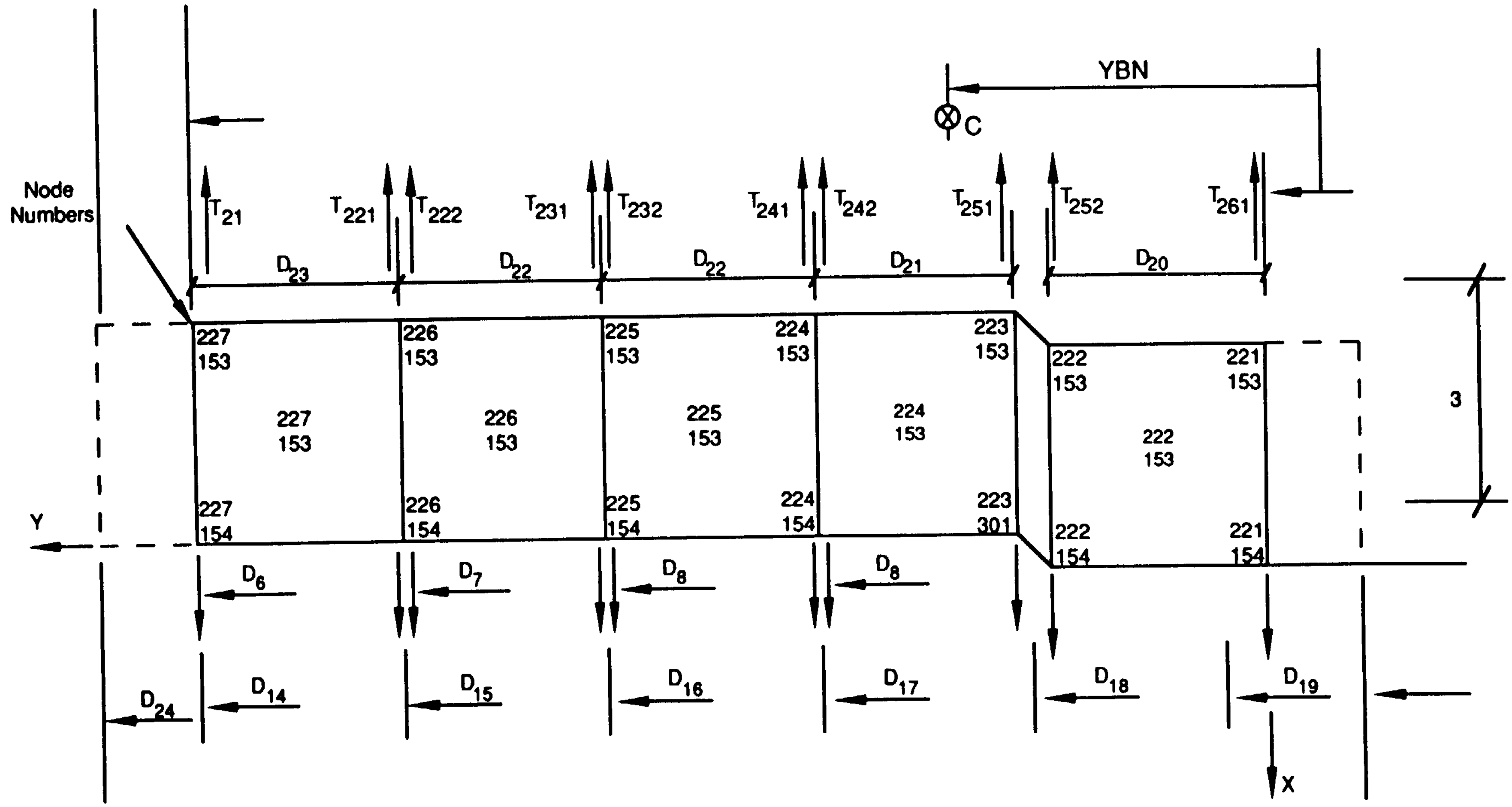


Figure 19. Cross-sectional properties and grid forces for critical section of case D (from ref. 1).

Table 5. Case D: Probabilistic input parameters.

Parameters	Nominal	Mean/Nom	Est. Cov.	Bias	Cov. Bias
T1	0.430	1.15	0.0400	1.01	0.05
D1	0.515	1.15	0.0042	1.01	0.05
D2	0.971	1.15	0.0042	1.01	0.05
D3	0.421	1.15	0.0042	1.01	0.05
D4	1.366	1.15	0.0042	1.01	0.05
D5	0.964	1.15	0.0042	1.01	0.05
D6	0.325	1.15	0.0042	1.01	0.05
D7	6.940	1.15	0.0042	1.01	0.05
D8	5.820	1.15	0.0042	1.01	0.05
D9	4.470	1.15	0.0042	1.01	0.05
D10	3.395	1.15	0.0042	1.01	0.05
D11	2.123	1.15	0.0042	1.01	0.05
D12	1.328	1.15	0.0042	1.01	0.05
D13	0.163	1.15	0.0042	1.01	0.05
YBN	3.345	1.15	0.0042	1.01	0.05
D14	6.395	1.15	0.0042	1.01	0.05
D15	5.145	1.15	0.0042	1.01	0.05
D16	3.795	1.15	0.0042	1.01	0.05
D17	2.445	1.15	0.0042	1.01	0.05
D18	1.525	1.15	0.0042	1.01	0.05
D19	0.585	1.15	0.0042	1.01	0.05
D20	0.940	1.15	0.0042	1.01	0.05
D21	0.920	1.15	0.0042	1.01	0.05
D22	1.350	1.15	0.0042	1.01	0.05
D23	1.250	1.15	0.0042	1.01	0.05
Wm	2.150	1.15	0.0042	1.01	0.05
D24	7.200	1.15	0.0042	1.01	0.05
Wmin	1.740	1.15	0.0042	1.01	0.05
T21	10,252.000	1.0	0.060	0.95	0.05
T221	17,205.000	1.0	0.060	0.95	0.05
T222	14,139.000	1.0	0.060	0.95	0.05
T231	15,936.000	1.0	0.060	0.95	0.05
T232	15,530.000	1.0	0.060	0.95	0.05

Table 5. Case D: Probabilistic input parameters (continued).

Parameters	Nominal	Mean/Nom	Est. Cov.	Bias	Cov. Bias
T241	8,955.000	1.0	0.060	0.95	0.05
T242	11,708.000	1.0	0.060	0.95	0.05
T251	3,473.000	1.0	0.060	0.95	0.05
T252	13,460.000	1.0	0.060	0.95	0.05
T261	4,146.000	1.0	0.060	0.95	0.05
Fy1	9,298.000	1.0	0.060	0.95	0.05
Fxy1	–3,531.000	1.0	0.060	0.95	0.05
Fxy2	–6,824.000	1.0	0.060	0.95	0.05
Fxy3	–11,864.000	1.0	0.060	0.95	0.05
Fxy4	–11,186.000	1.0	0.060	0.95	0.05
Fxy5	–9,245.000	1.0	0.060	0.95	0.05
Mx1	–82.100	1.0	0.060	0.95	0.05
Mx2	91.500	1.0	0.060	0.95	0.05
Mx3	–39.600	1.0	0.060	0.95	0.05
Mx4	–230.000	1.0	0.060	0.95	0.05
Mx5	–625.000	1.0	0.060	0.95	0.05
My1	–440.000	1.0	0.060	0.95	0.05
T2H	0.250	1.15	0.0400	1.01	0.05
FSY	1.100	1.0	0.01	1.00	0.00
FSU	1.400	1.0	0.01	1.00	0.00
SIGy	150.000	1.055	0.022	1.00	0.00
SIGu	180.000	1.055	0.022	1.00	0.00

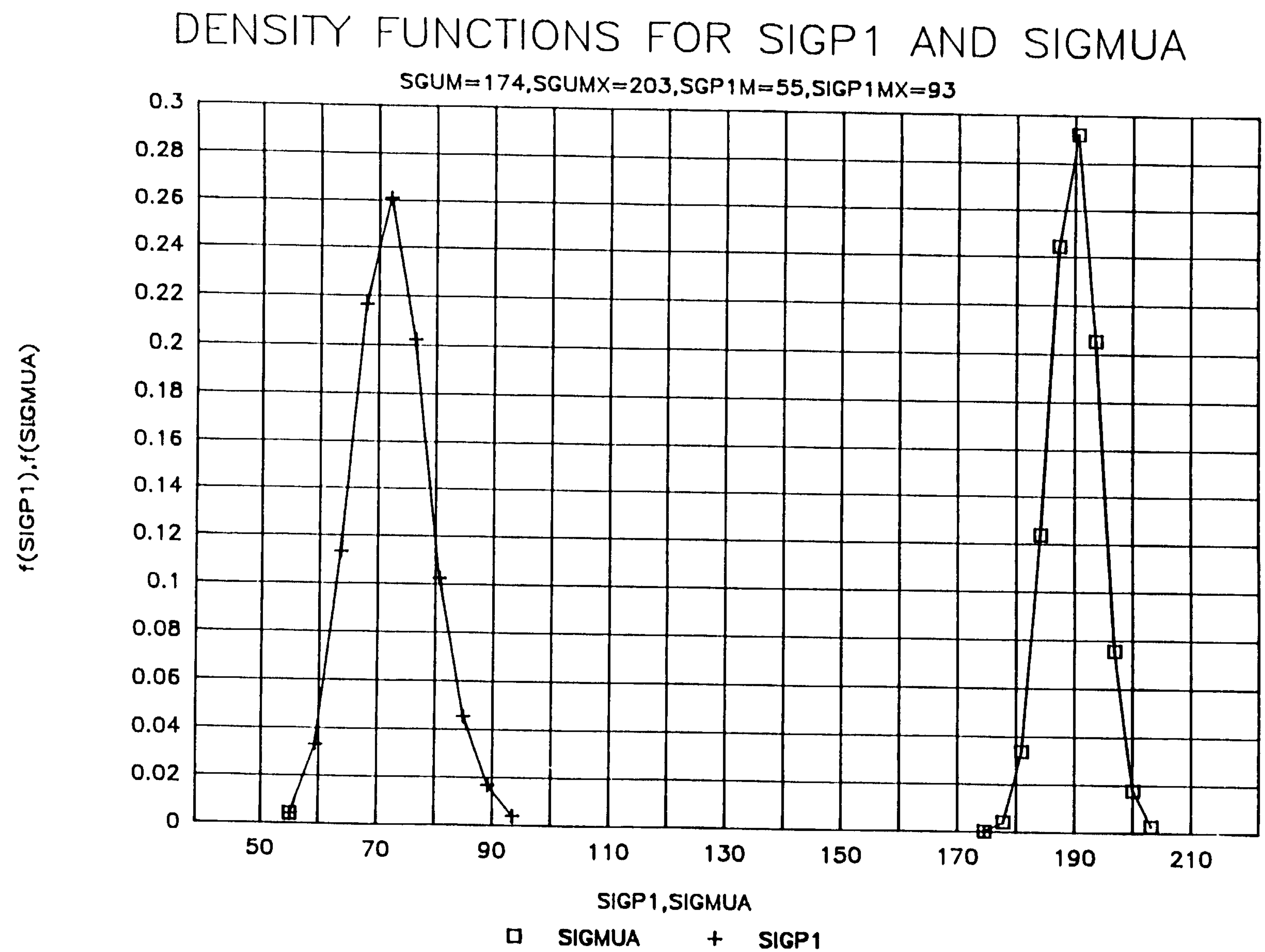


Figure 20. Warner diagram for case D.

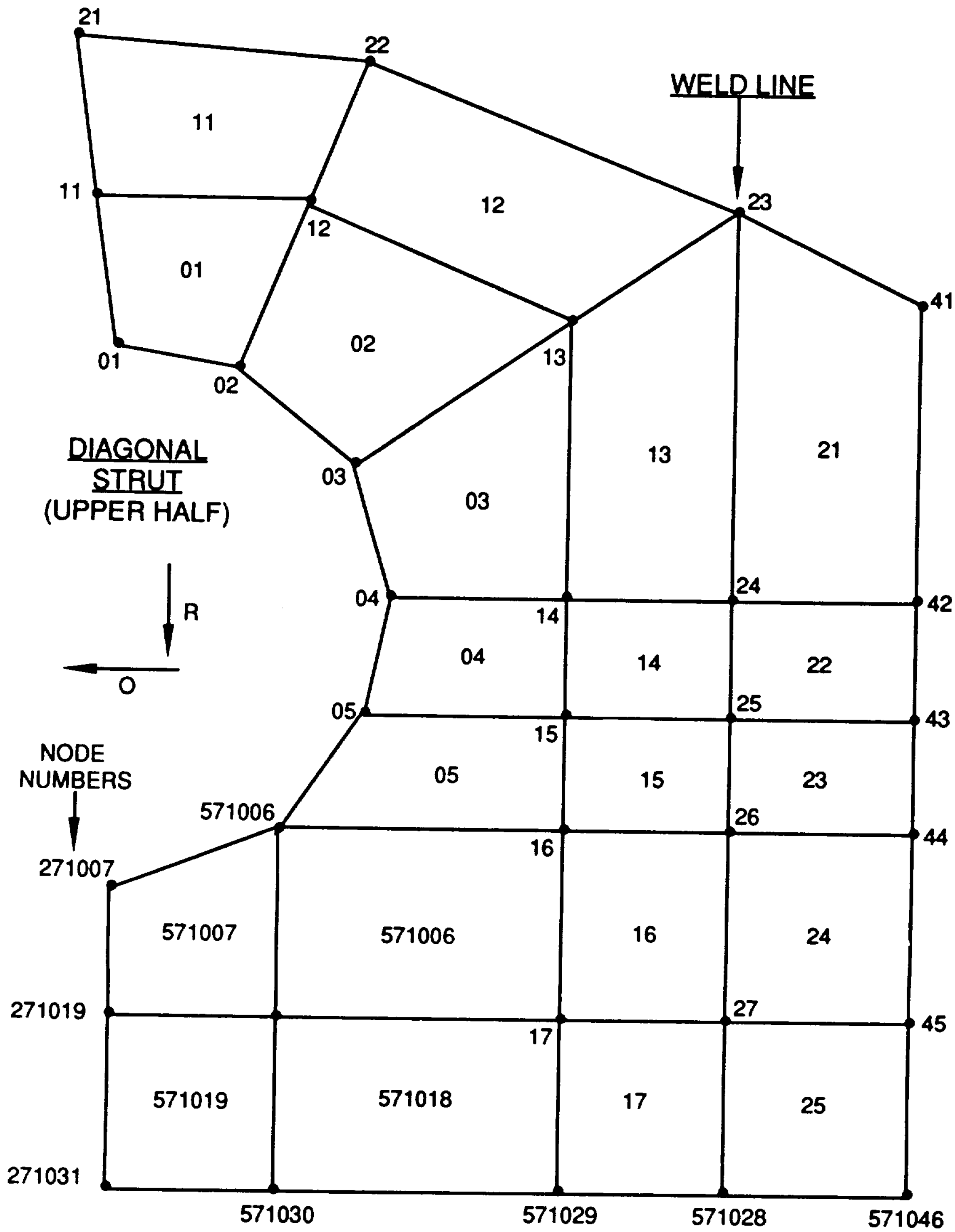


Figure 21. H-fitting lug section (case E) (from ref. 1).

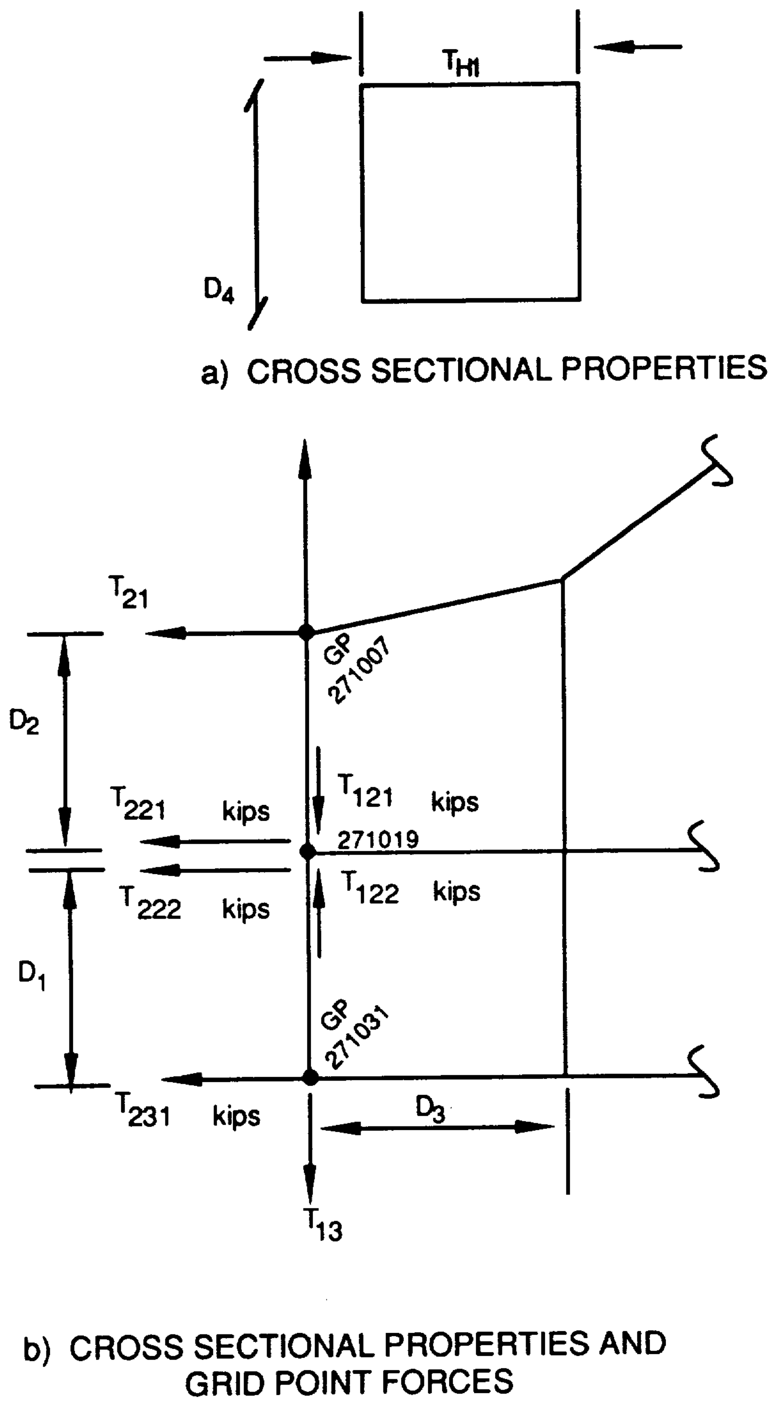


Figure 22. Cross-sectional properties and grid point forces for critical section of case E (from ref. 1).

Table 6. Case E: Probabilistic input parameters.

Parameters	Nominal	Mean/Nom	Est. Cov.	Bias	Cov. Bias
TH1	1.440	1.15	0.0400	1.01	0.05
D1	0.760	1.15	0.0042	1.01	0.05
D2	0.770	1.15	0.0042	1.01	0.05
D3	0.990	1.15	0.0042	1.01	0.05
D4	1.499	1.15	0.0042	1.01	0.05
T11	–41.300	1.0	0.06	0.95	0.05
T21	30.060	1.0	0.06	0.95	0.05
T121	9.930	1.0	0.06	0.95	0.05
T122	–6.720	1.0	0.06	0.95	0.05
T221	58.430	1.0	0.06	0.95	0.05
T222	53.520	1.0	0.06	0.95	0.05
T13	0.780	1.0	0.06	0.95	0.05
T231	72.500	1.0	0.06	0.95	0.05
Mx1	4.680	1.0	0.06	0.95	0.05
Mx2	0.140	1.0	0.06	0.95	0.05
My1	0.260	1.0	0.06	0.95	0.05
My2	3.920	1.0	0.06	0.95	0.05
P	214.510	1.0	0.06	0.95	0.05
FSY	1.100	1.0	0.01	1.00	0.00
FSU	1.400	1.0	0.01	1.00	0.00
SIGy	150.000	1.055	0.022	1.00	0.00
SIGu	180.000	1.055	0.022	1.00	0.00

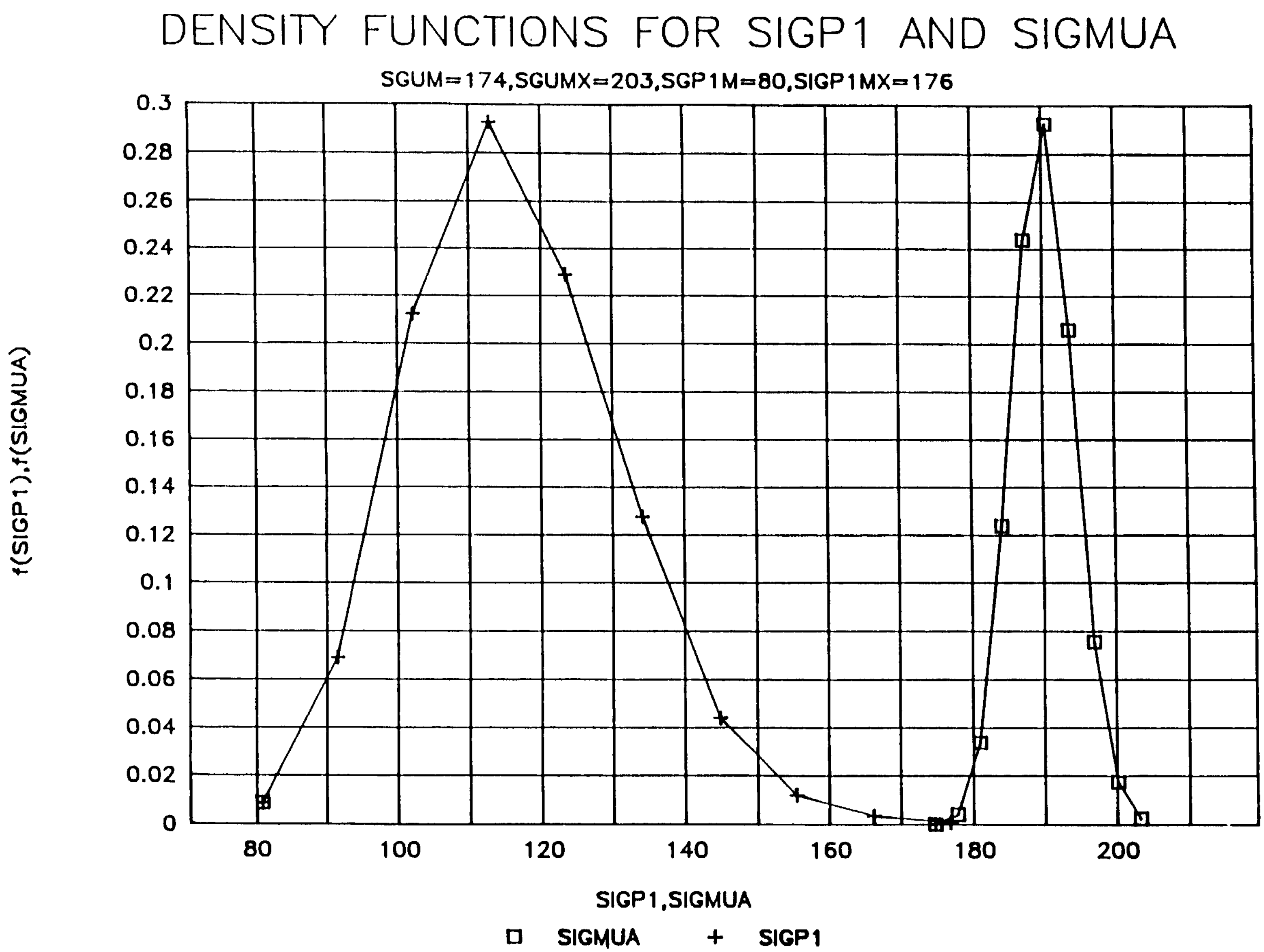


Figure 23. Warner diagram for case E.

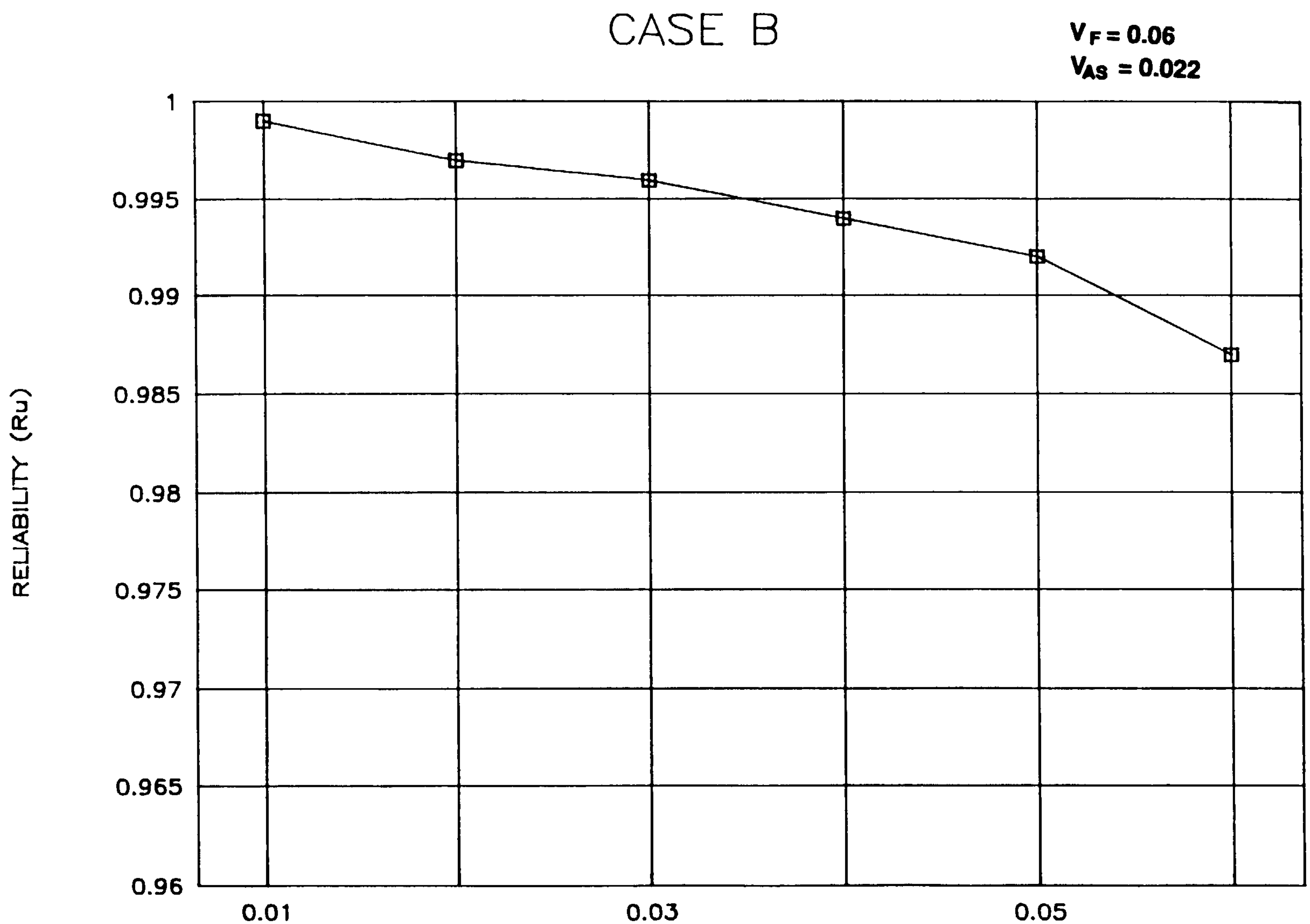


Figure 24. Effect of coefficient of variation of cross-sectional parameters (V_c).

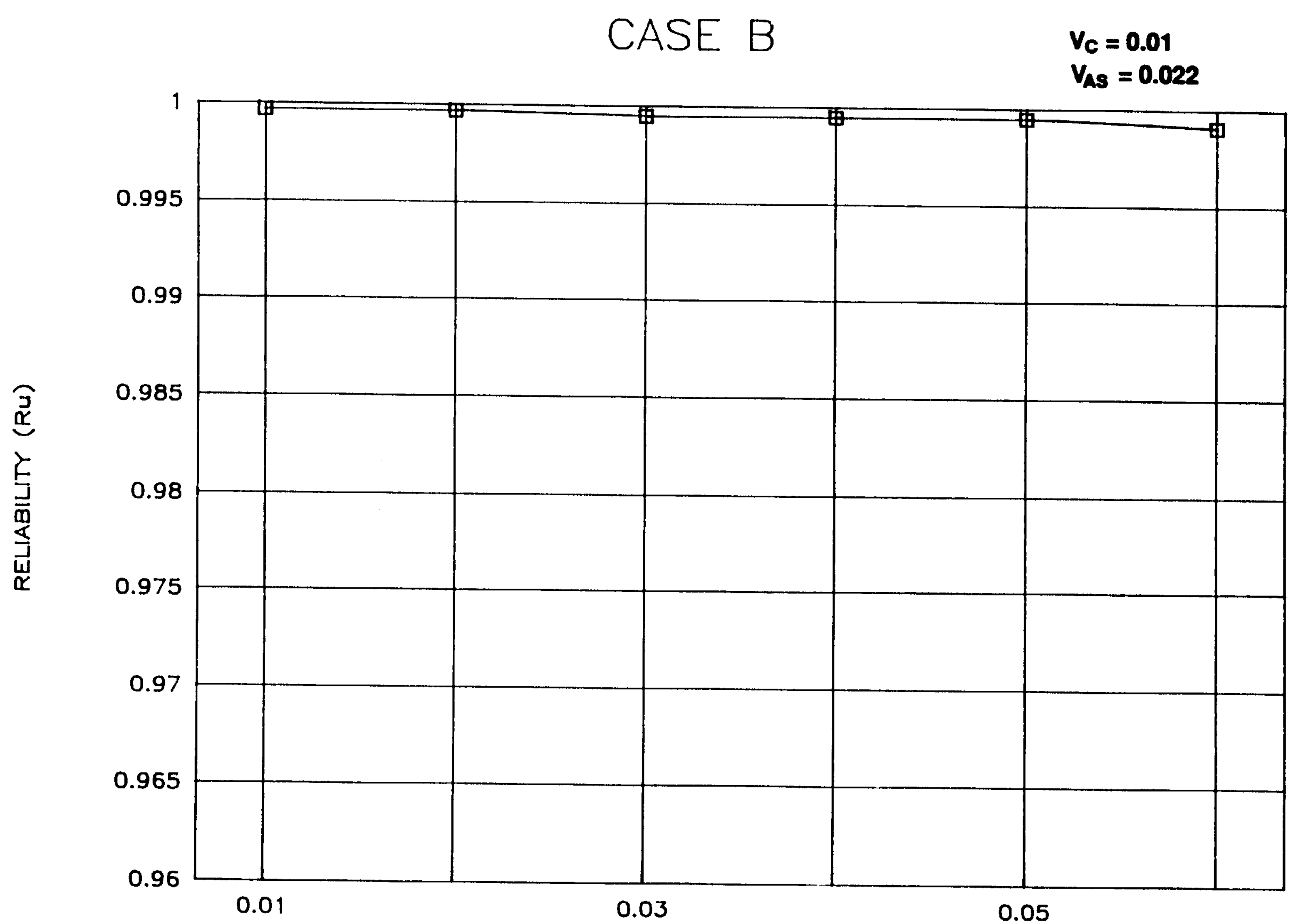


Figure 25. Effect of coefficient of variation of force parameters (V_f).

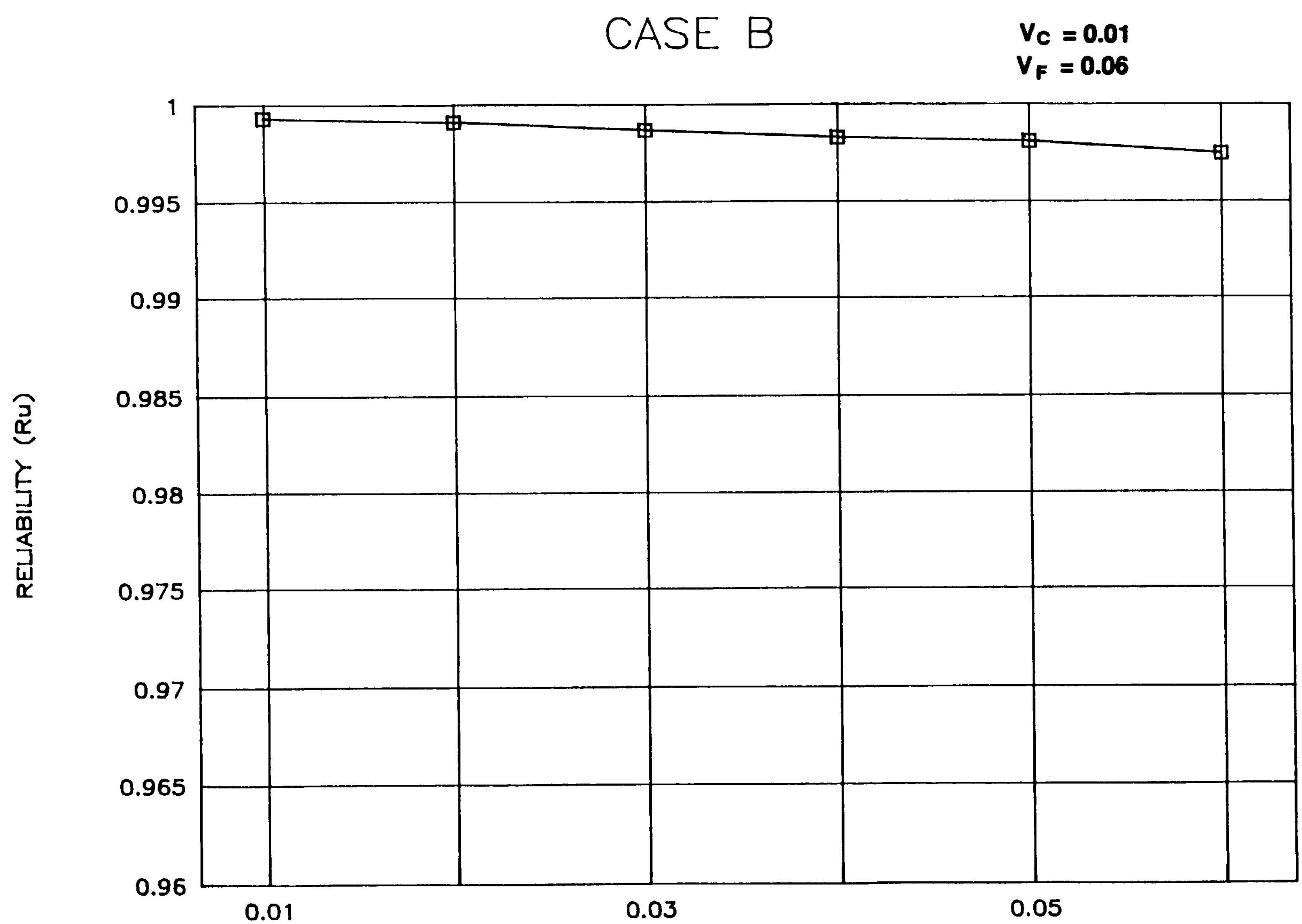


Figure 26. Effect of coefficient of variation of stress parameters (V_{AS}).

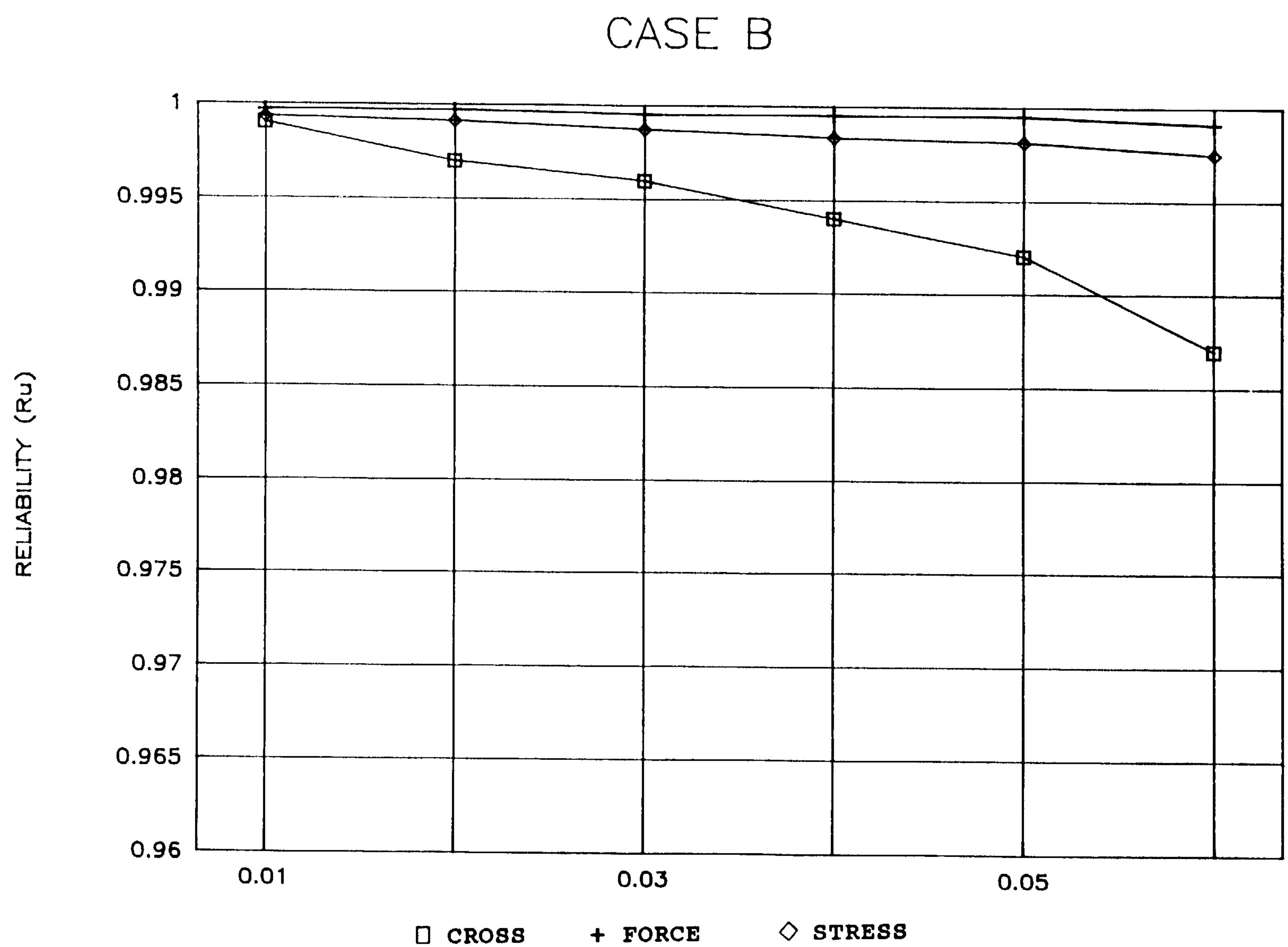


Figure 27. Combined effects of coefficient of variation of input parameters (V_c, V_f, and V_{AS}).

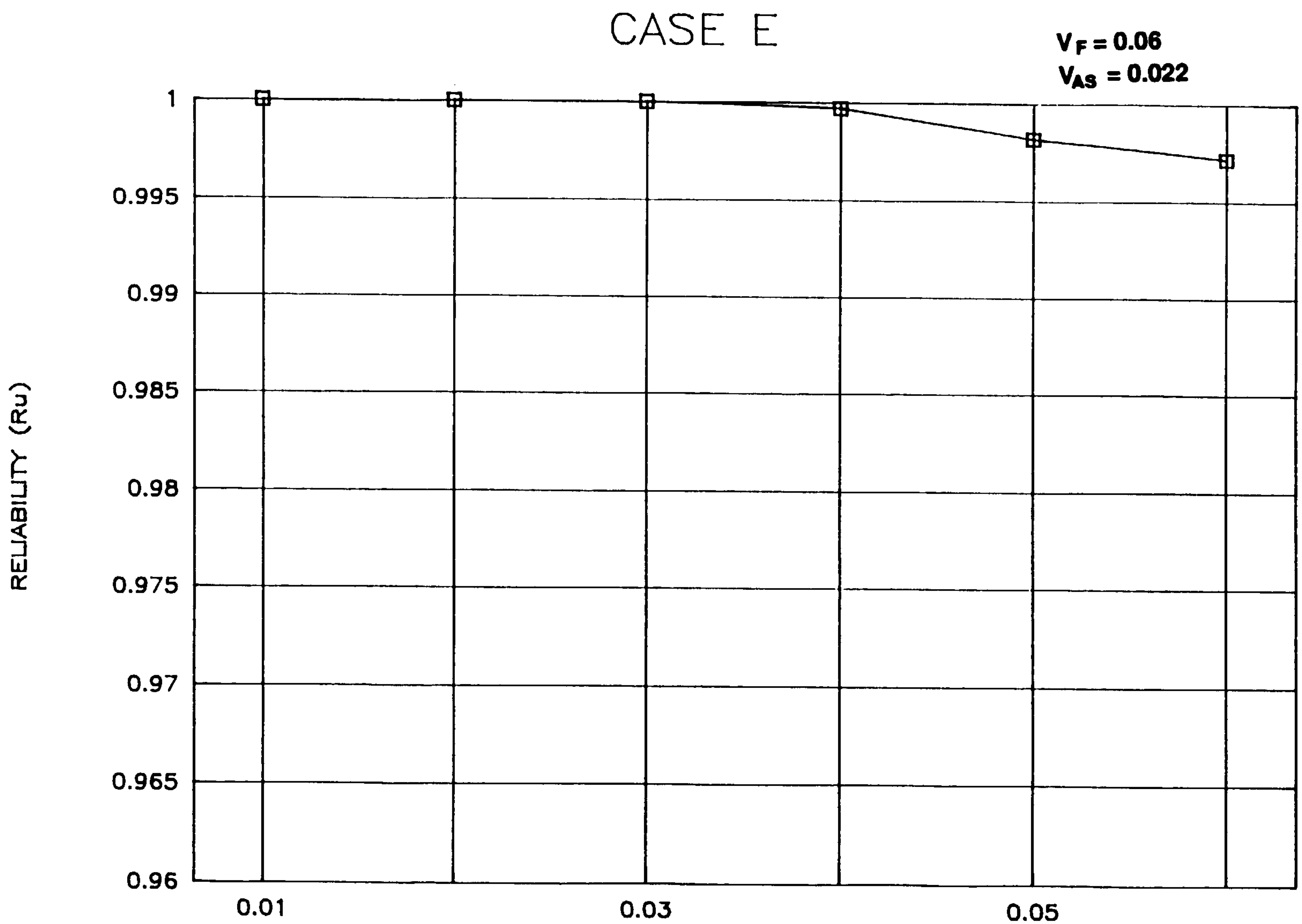


Figure 28. Effect of coefficient of variation of cross-sectional parameters (V_C).

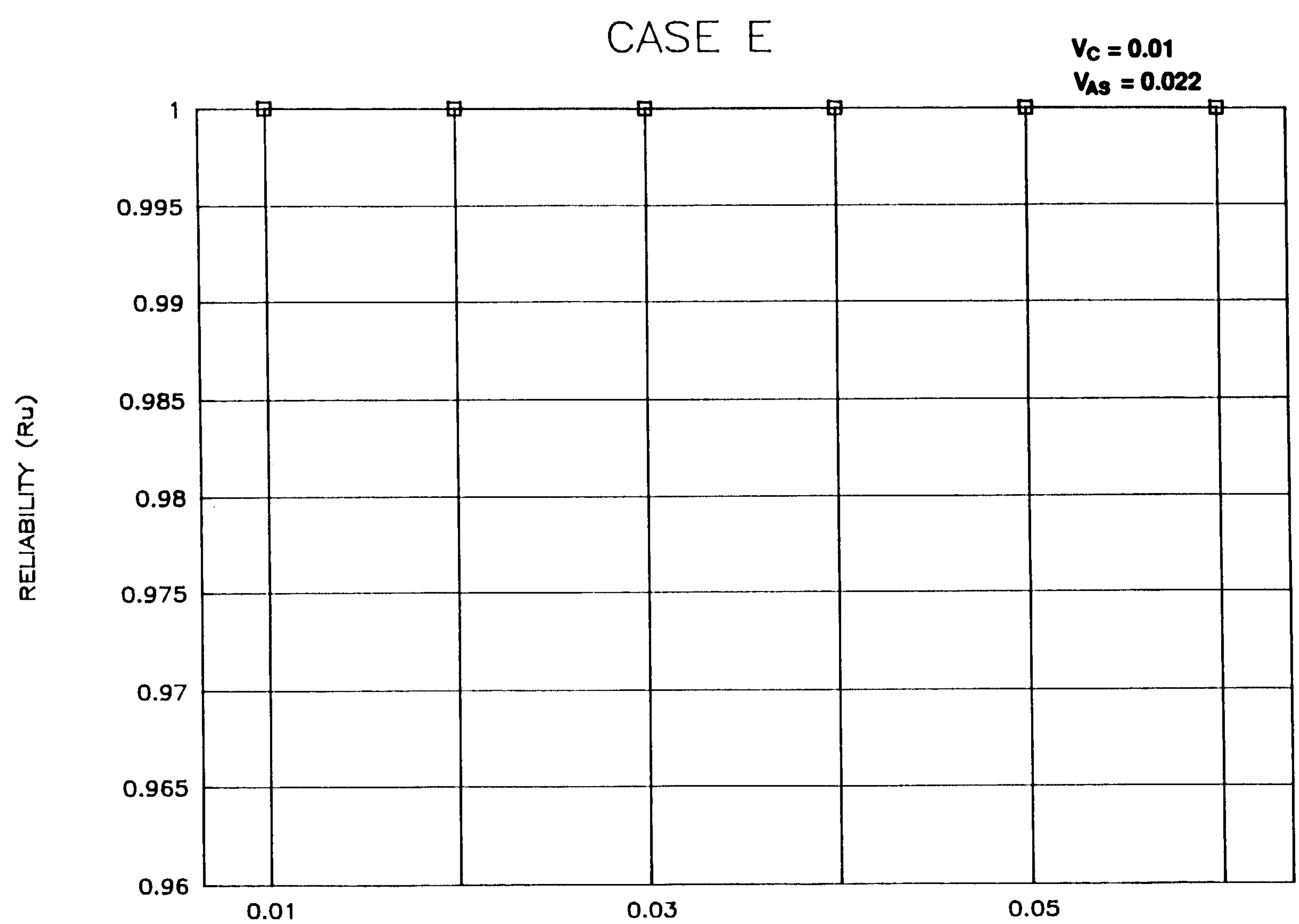


Figure 29. Effect of coefficient of variation of force parameters (V_F).

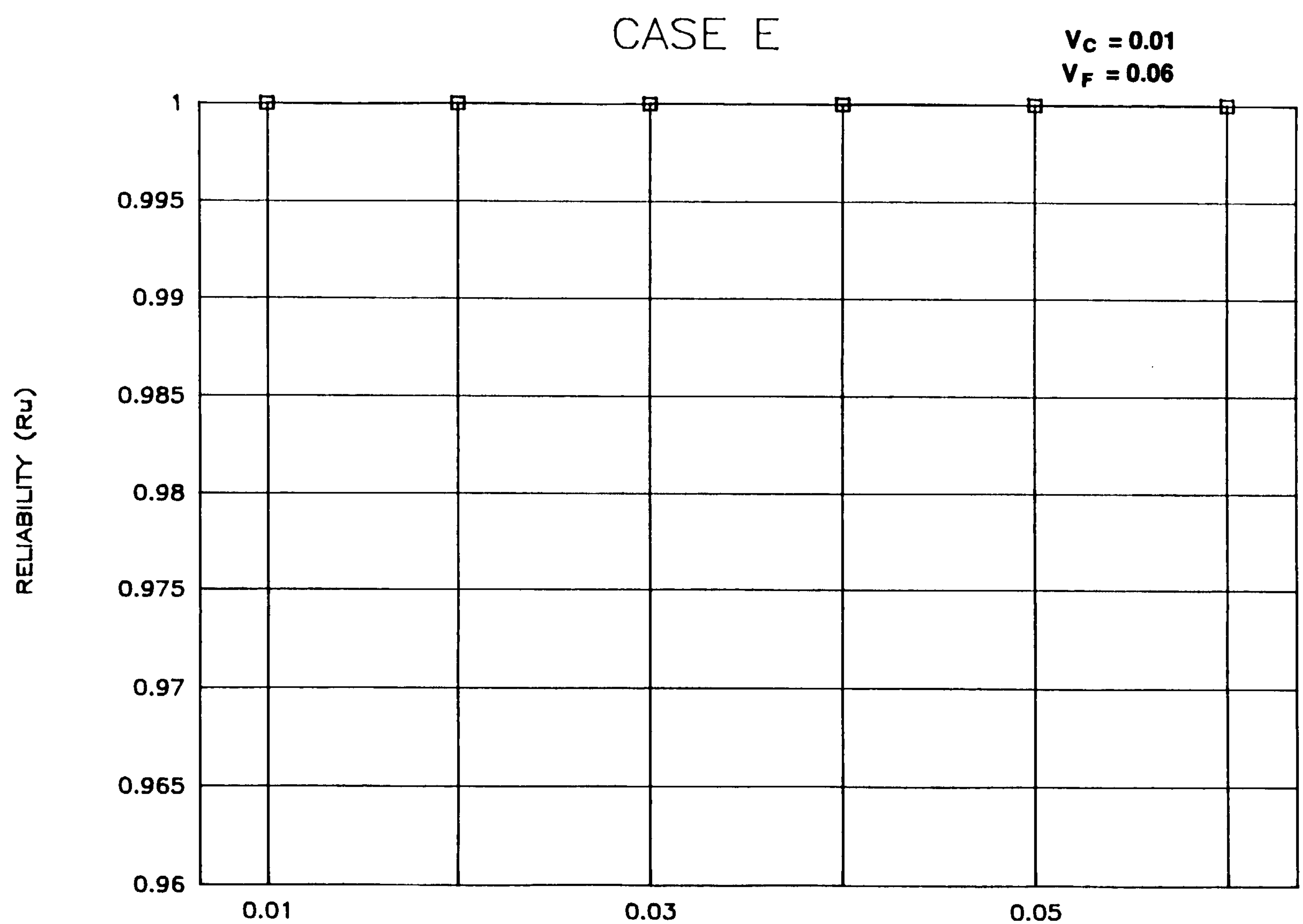


Figure 30. Effect of coefficient of variation of stress parameters (V_{AS}).

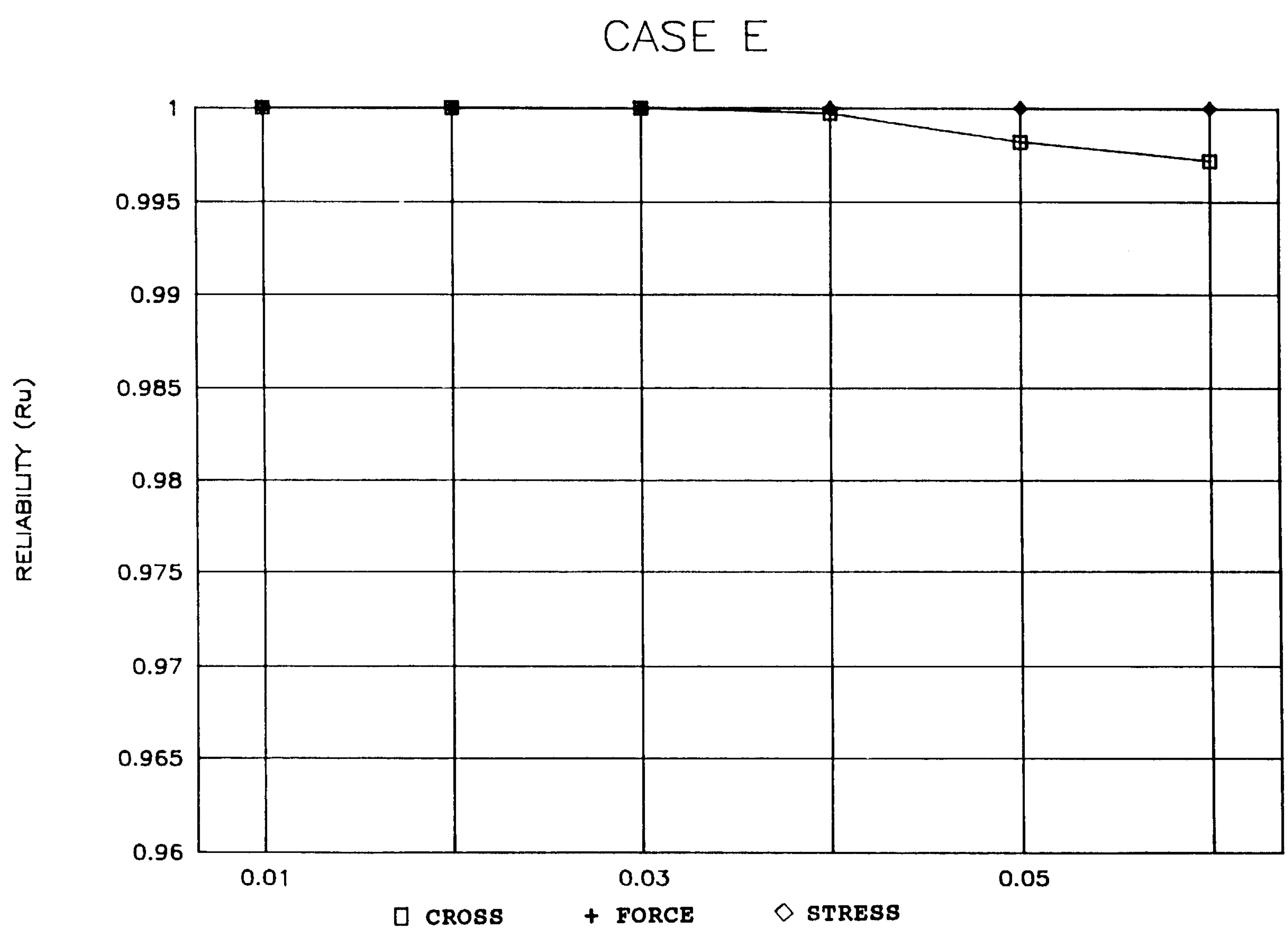


Figure 31. Combined effect of coefficient of variation of input parameters.

allowable stress parameters (C_{mns}). To be specific, C_{mnc} stands for coefficient of mean to nominal for cross-sectional parameters, C_{mnf} stands for coefficient of mean to nominal for force parameters, and C_{mns} stands for coefficient of variation of mean to nominal for stress parameters. The basic values considered are C_{mnc} = 1.15, C_{mnf} = 1.0, and C_{mns} 1.055. These results are shown in figures 32 to 35 for case B and in figures 36 to 39 for case E.

The effect of the sample size of random numbers on the reliability of the ETA ring is also studied by varying the number of random numbers from 5,000 to 20,000 and using varying reduction techniques as well as direct Monte Carlo simulation. The effect is found to be insignificant so far as the reliability values of the ETA ring are concerned.

VIII. DISCUSSION OF RESULTS AND CONCLUSIONS

It can be seen from various cases discussed earlier in section VI that the coefficient of variation of the principal stress is greater than or equal to 0.10, and, in case D, it is as high as 0.18 even though the input coefficient of variation is not very high. This is because the expression for principal stress (equation (27)) is highly nonlinear and is a function of other dependent variables like normal and shear stresses. It should be noted that these stresses are again nonlinear functions of some dependent and independent variables. The cumulative effect of all this is reflected in the coefficient of variation of one of the limit state variables of principal stress. This in effect affects the reliability levels.

Looking at figures 27 and 31, it can be seen that the V_c has maximum effect on reliability for cases B and E. Similarly, looking at figures 35 and 39, it can be seen that C_{mnc} has maximum effect on reliability for cases B and E. Another interesting point to be noted from these figures is that the optimal value (the common intersection point) of C_{mn} is approximately 1.1 for both cases B and E.

On the whole, the technique of Monte Carlo simulation with and without variance reduction techniques has been used successfully for evaluation of reliability levels of the ETA ring for the stress limit state. The cross-sectional parameters are found to be most critical as their effect on reliability is found to be maximum. Reliability levels in general are found to be high and appear to be supportive of the deterministic results and with a higher safety reserve.

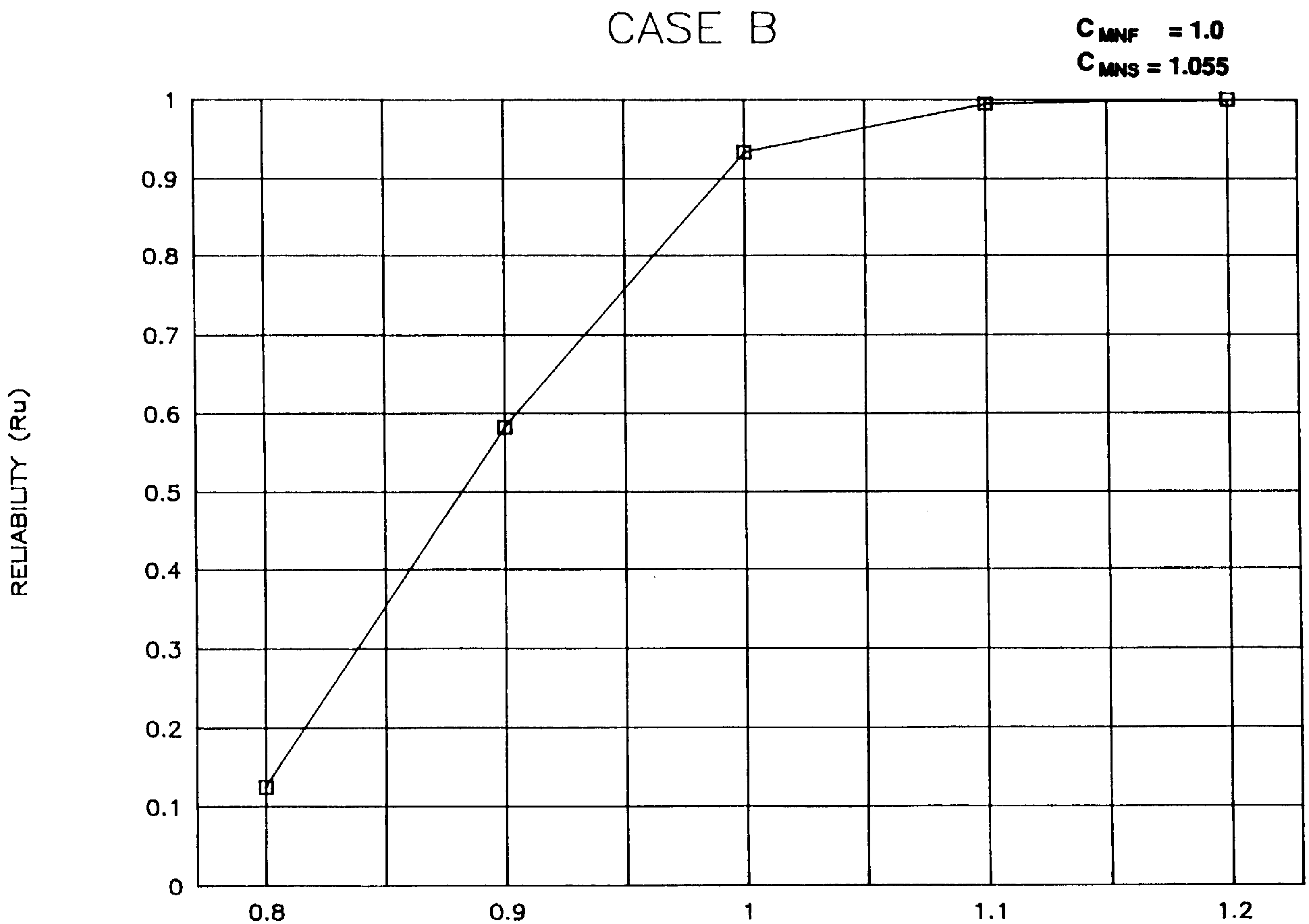


Figure 32. Effect of coefficient of mean/nominal for cross-sectional parameters (C_{MNC}).

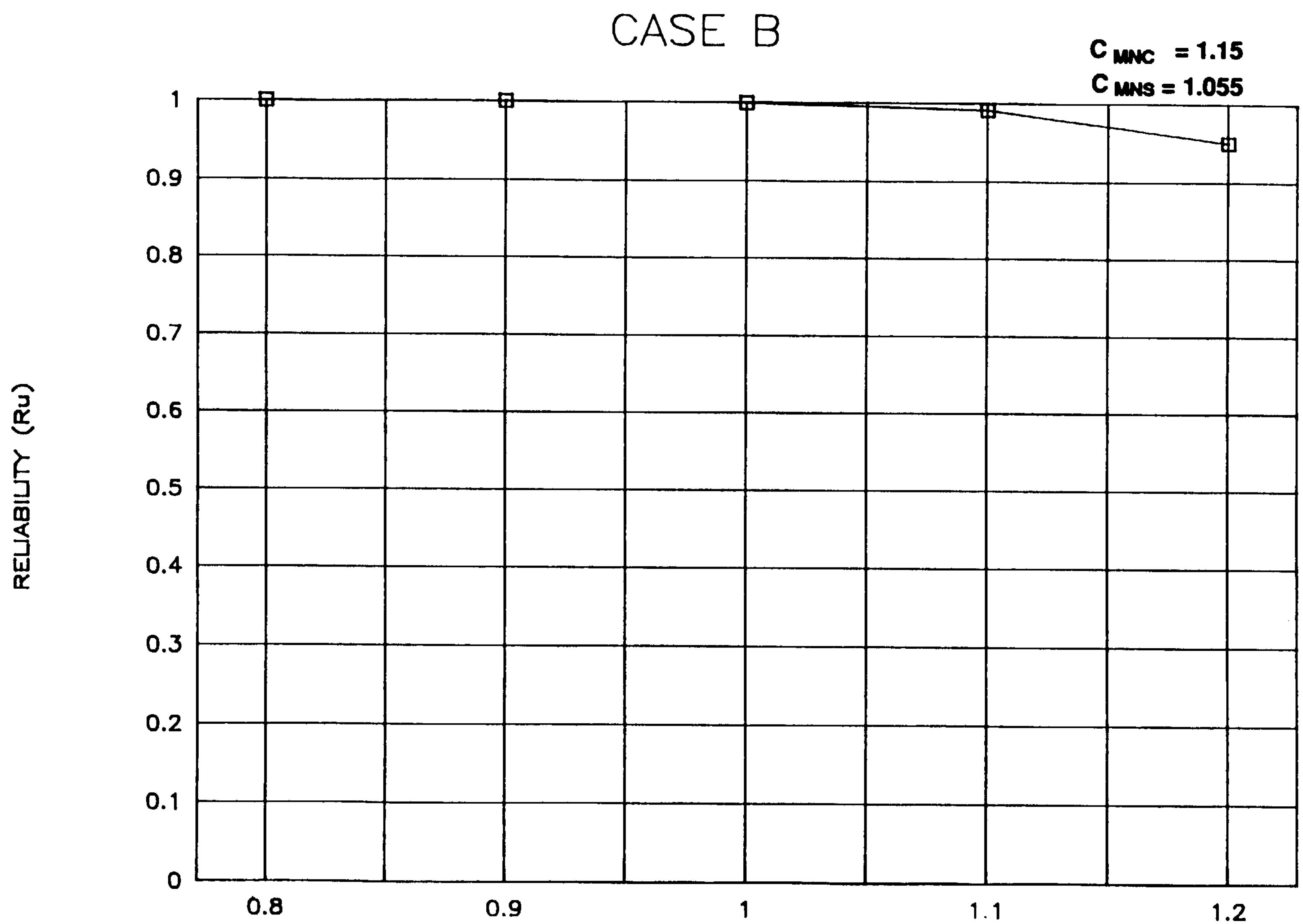


Figure 33. Effect of coefficient of mean/nominal for force parameters (C_{MNF}).

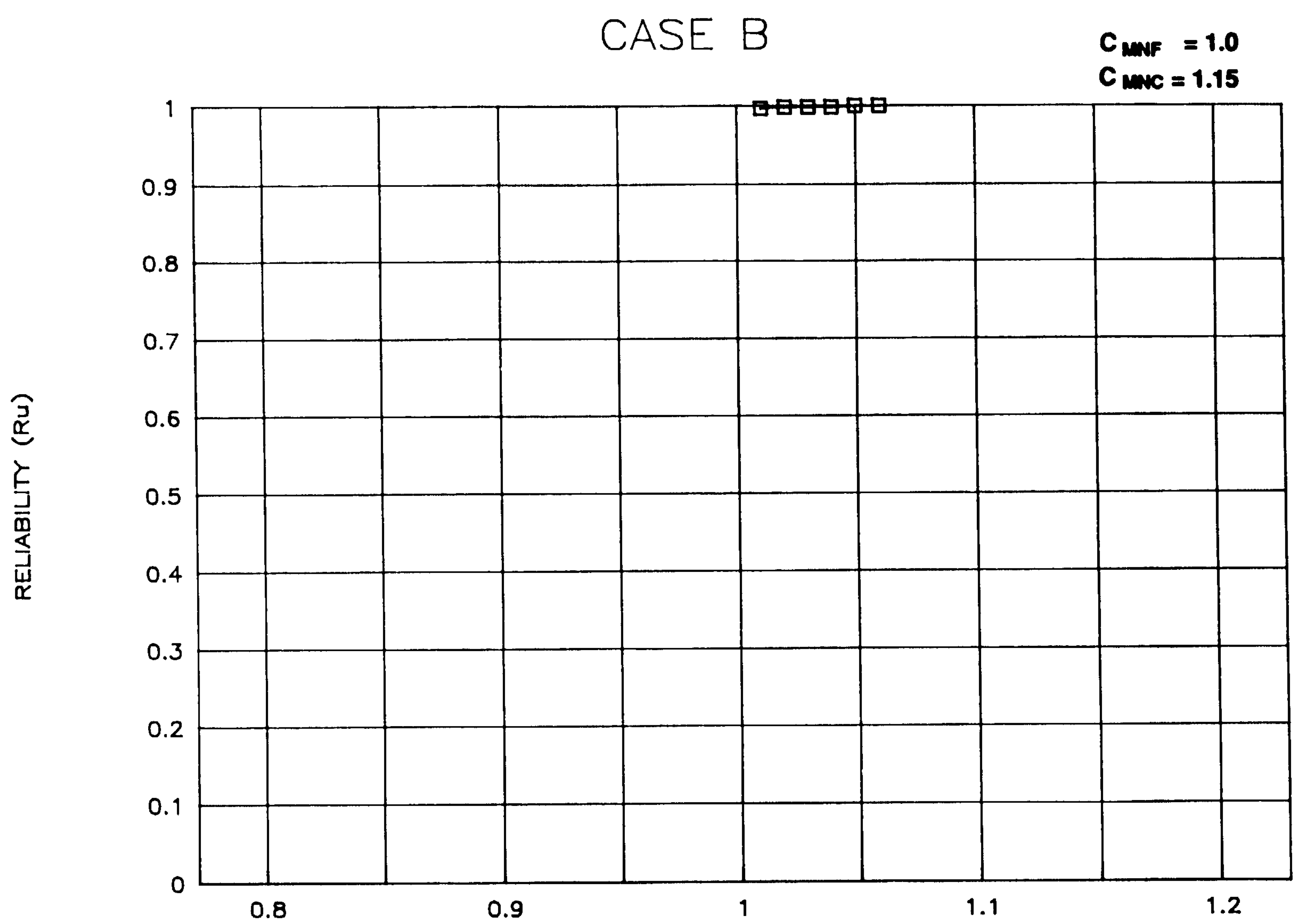


Figure 34. Effect of coefficient of mean/nominal for stress parameters (C_{MNS}).

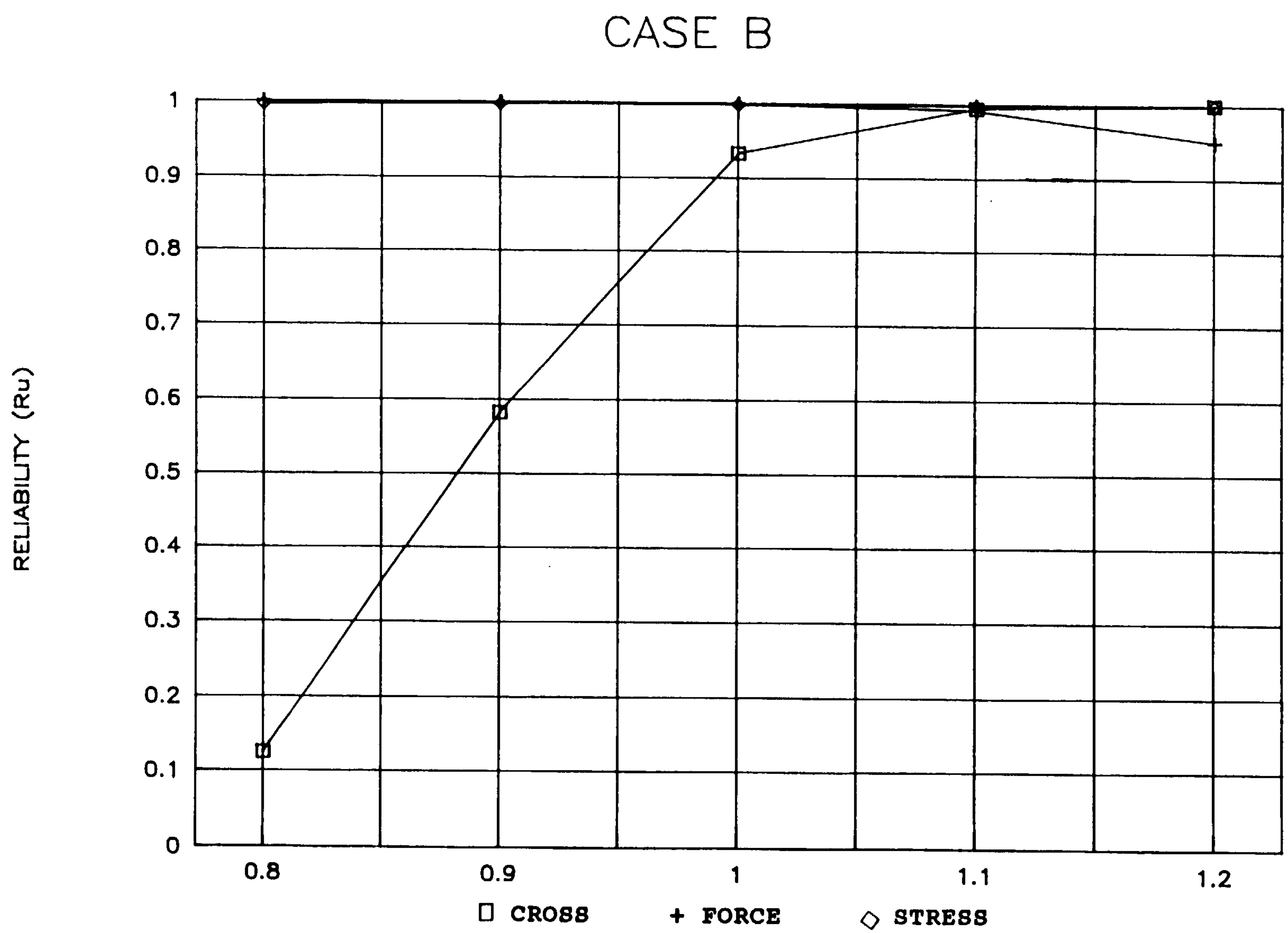


Figure 35. Combined effect of coefficient of mean/nominal parameters (C_{MNC}, C_{MNF}, and C_{MNS}).

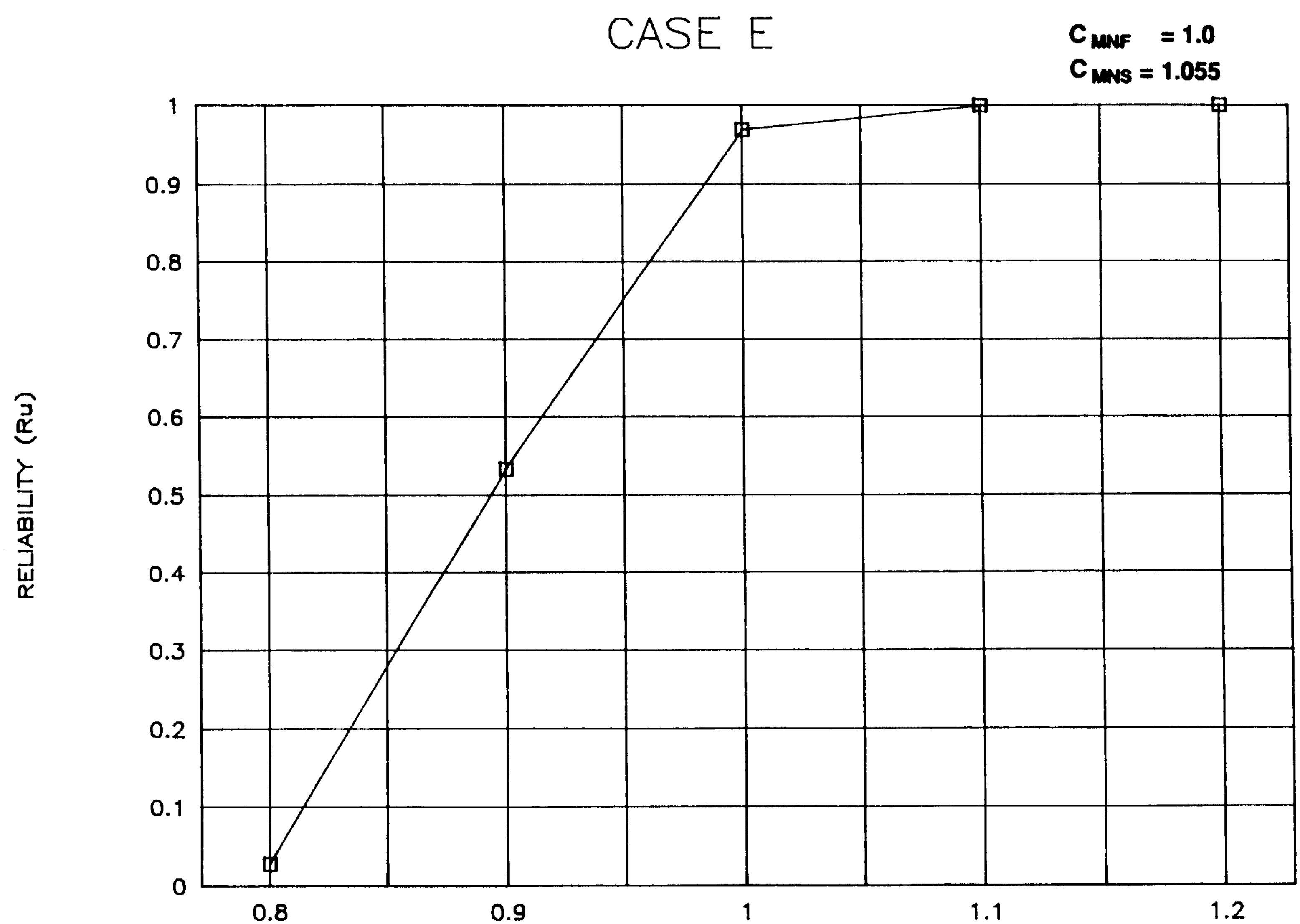


Figure 36. Effect of coefficient of mean/nominal for cross-sectional parameters (C_{MNC}).

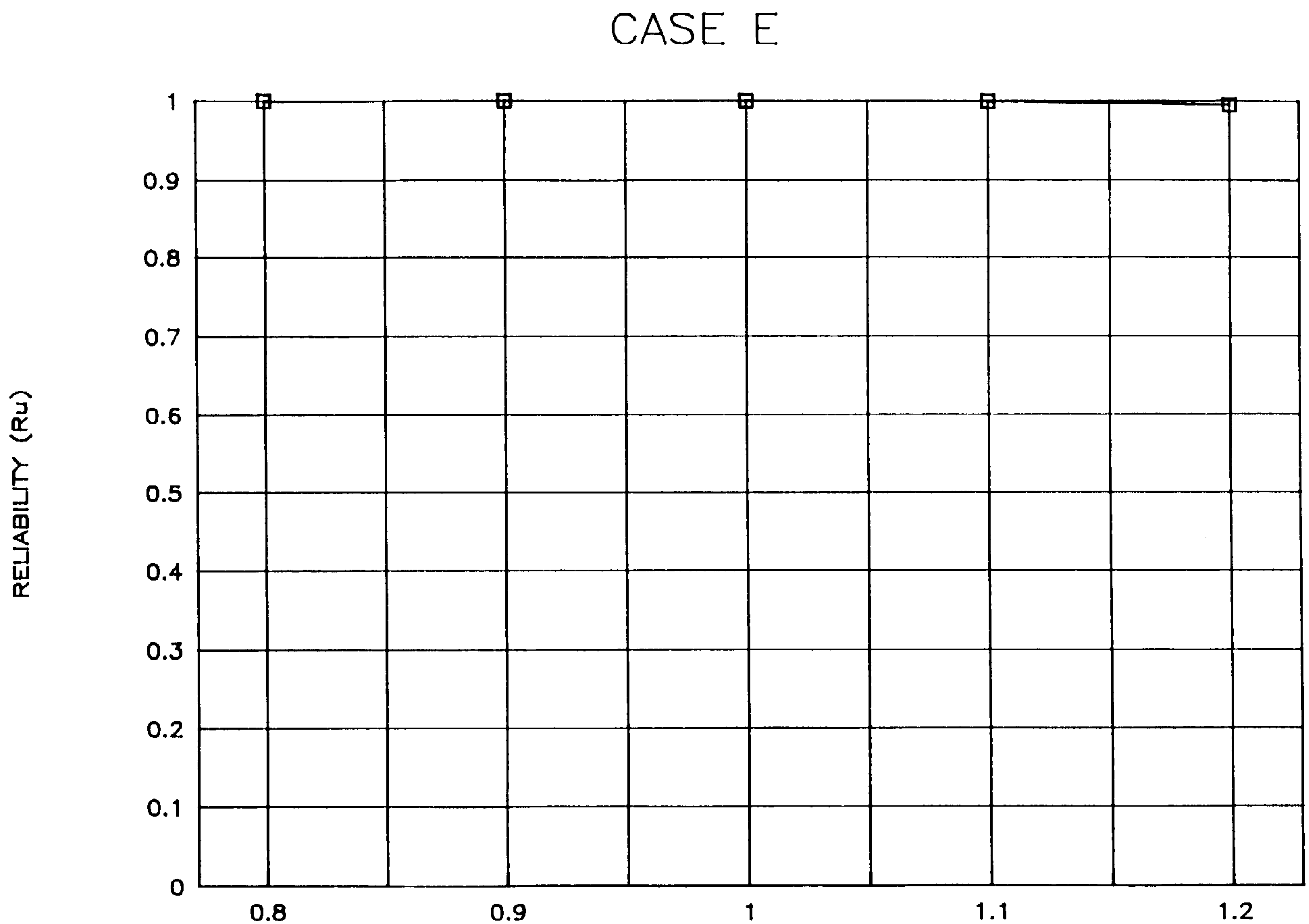


Figure 37. Effect of coefficient of mean/nominal for force parameters (C_{MNF}).

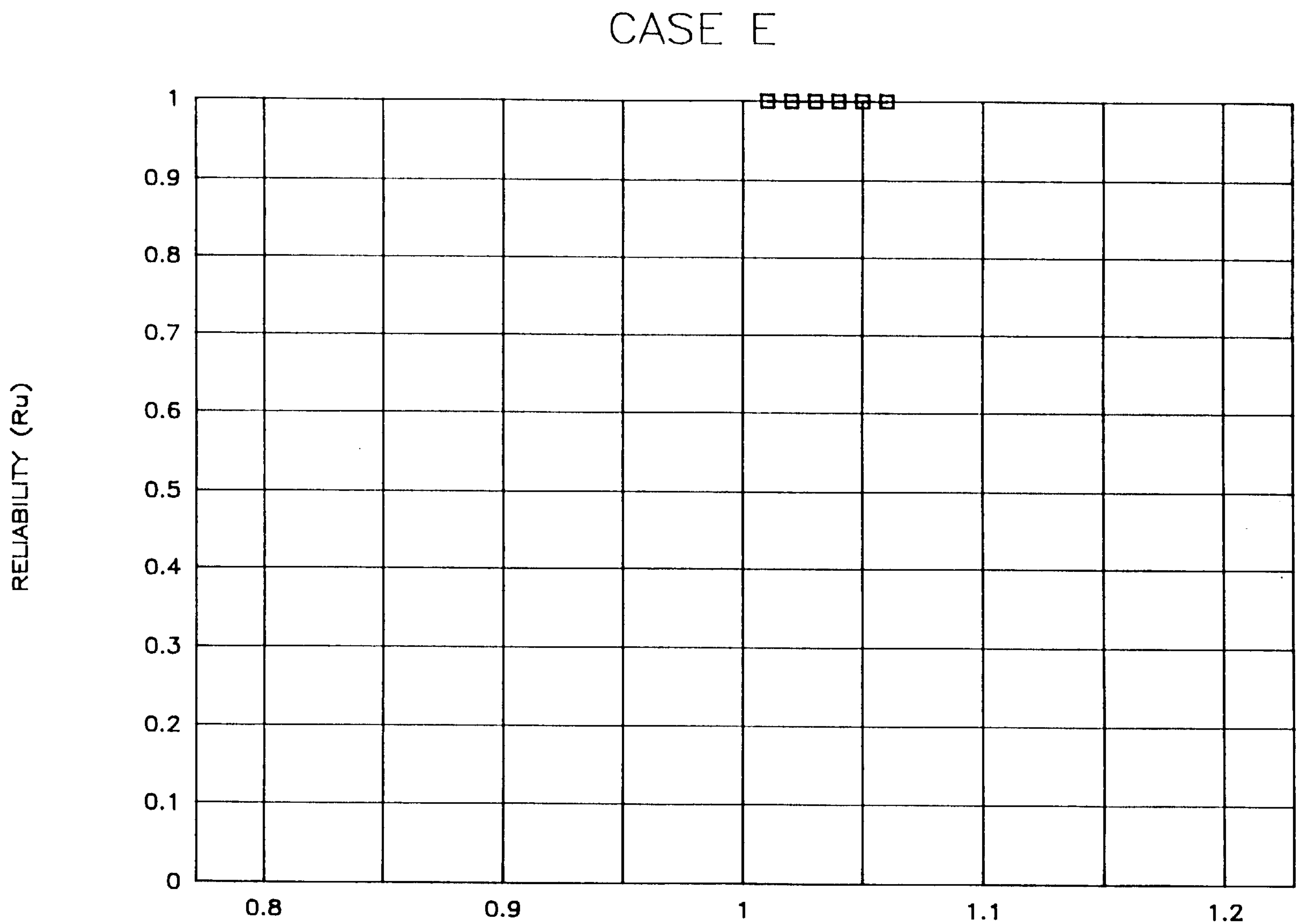


Figure 38. Effect of coefficient of mean/nominal for stress parameters (C_{MNS}).

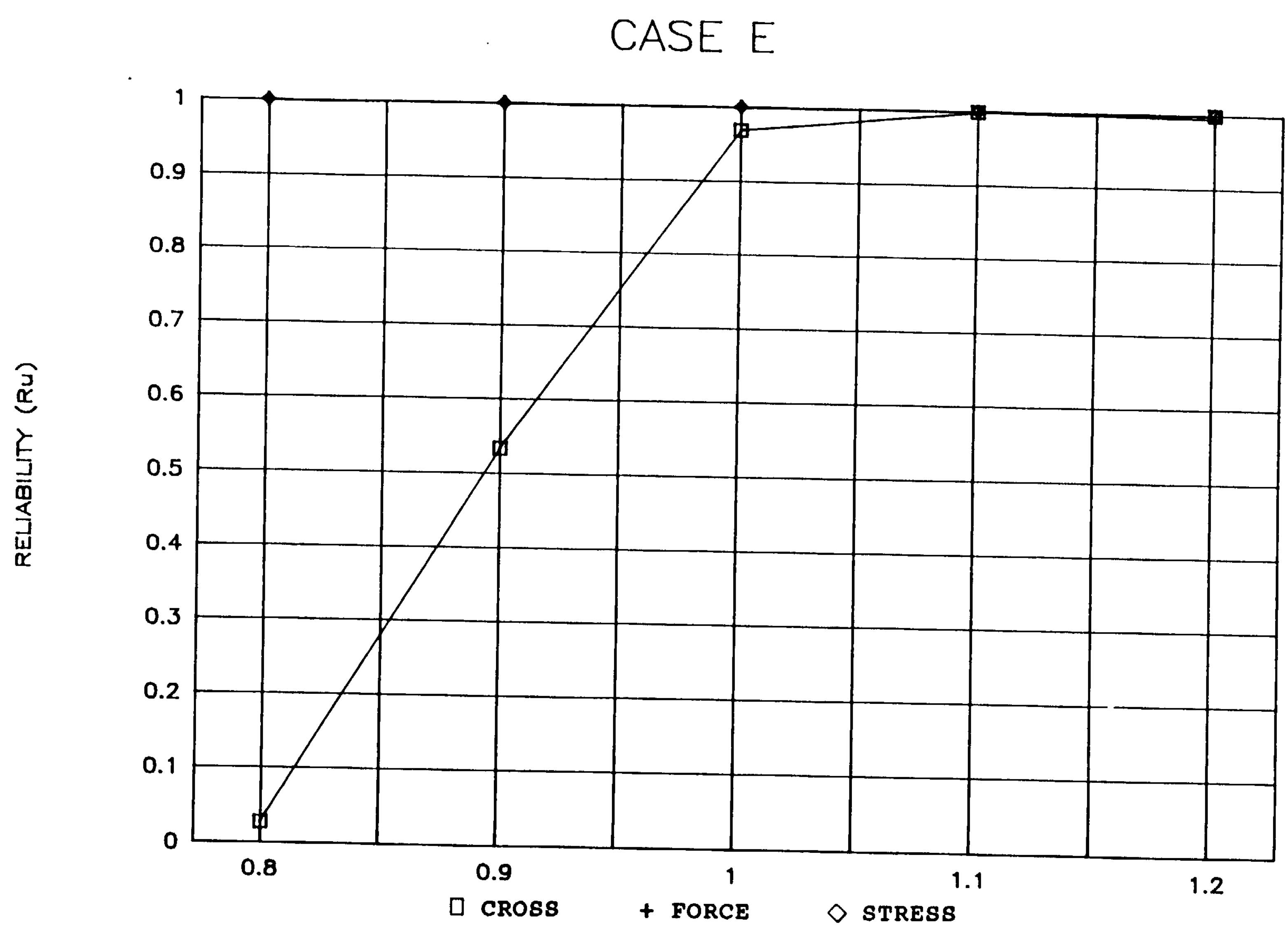


Figure 39. Combined effect of mean/nominal parameters (C_{MNC}, C_{MNF}, and C_{MNS}).

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APPROVAL

AN IN-DEPTH PROBABILISTIC STUDY OF EXTERNAL TANK ATTACH RING

By F. Pizzano and C. Putcha

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

J.H. EHL
Director, Safety and Mission Assurance

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